Scheduling in Multi-Channel Wireless Networks with Limited Information

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Abstract

The availability of multiple orthogonal channels in a wireless network can potentially lead to substantial performance improvement by alleviating contention and interference. However, this also gives rise to non-trivial channel coordination issues. The situation is exacerbated by variability in the achievable data-rates across channels and links. Thus, scheduling in such networks may require substantial information-exchange and lead to non-negligible overhead. This provides a strong motivation for the study of scheduling algorithms that can operate with limited information, while still providing acceptable worst-case performance guarantees. In this paper, we make an effort in this direction, by examining the scheduling implications of multiple channels, and heterogeneity in channel-rates. We establish lower bounds on performance of a class of *maximal* schedulers, and describe a scheduler that require limited information-exchange between nodes. We first demonstrate that when the underlying scheduling mechanism is "imperfect", the presence of multiple orthogonal channels can help alleviate the detrimental impact of the imperfect scheduler, and yield a significantly better efficiency-ratio in a wide range of network topologies. We then establish performance bounds for a scheduler than can achieve good efficiency-ratios in the presence of channels with heterogeneous rates without requiring explicit exchange of queue-information. Our results indicate that it may be possible to achieve a desirable trade-off between performance and information.

I. Introduction

Appropriate scheduling policies are of utmost importance in achieving good throughput characteristics in a wireless network. The seminal work of Tassiulas and Ephremides yielded a *throughput-optimal* scheduler, which can schedule all "feasible" traffic flows without resulting in unbounded queues [1]. However, such an optimal scheduler is difficult to implement in practice. Thus various imperfect scheduling strategies that trade-off throughput for simplicity have been proposed [2], [3], [4], [5] amongst others.

The availability of multiple orthogonal channels in a wireless network can potentially lead to substantial performance improvement by alleviating contention and interference. However, this also gives rise to non-trivial channel coordination issues. The situation is exacerbated by variability in the achievable data-rates across channels and links. Computing an optimal schedule, even in a single-channel network, is almost always intractable both due to need for global information, and computational complexity. However, imperfect schedulers requiring limited *local* information can typically be designed, which provide acceptable worst-case (and typically much better average

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case) performance degradation compared to the optimal. In a multi-channel network, the local information exchange required by even an imperfect scheduler can be quite prohibitive, as information may be needed on a per-channel basis. For instance, Lin and Rasool [6] have described a scheduling algorithm for multi-channel multi-radio wireless networks that requires information about *per-channel* queues at all interfering links.

This provides a strong motivation for the study of scheduling algorithms that can operate with limited information, while still providing acceptable worst-case performance guarantees. In this paper, we make an effort in this direction, by examining the scheduling implications of multiple channels, and heterogeneity in channel-rates. We establish lower bounds on performance of a class of *maximal* schedulers, and describe some schedulers that require limited information-exchange between nodes. Some of the bounds presented here improve on bounds developed in past work [6].

We begin by analyzing the performance of a centralized greedy maximal scheduler. A lower bound for this scheduler was established in [6], which is tight in the sense that there exists a network topology in which the upper bound matches this lower bound. However, in a large variety of network topologies, the lower bound can be quite loose. Thus is particularly true for networks with single interface nodes. We establish an alternative bound that is tighter in a range of topologies. *Our results indicate that when the underlying scheduling mechanism is imperfect, the presence of multiple orthogonal channels can help alleviate the impact of the imperfect scheduler, and yield a significantly better efficiency-ratio in a wide range of scenarios..*

We then consider the possibility of achieving efficiency-ratio comparable to the centralized greedy maximal scheduler using a simpler scheduler that works with limited information. We establish results for a class of maximal schedulers coupled with local queue-loading rules that do not require queue-information from interfering nodes.

On a related note, cross-layer resource allocation in multi-channel wireless networks has been considered in [7].

II. Preliminaries

We consider a multi-hop wireless network. For simplicity, we will limit much of our discussion to nodes equipped with a single interface (or single radio) capable of tuning to any one available channel at any given time. The interface may switch between channels if desired. The results presented in the paper can also be used to obtain results for the case when each node is equipped with multiple interfaces: we briefly discuss this issue.

The wireless network is viewed as a directed graph, with each directed link in the graph representing an available communication link. We model interference using a *conflict* relation between links. Two links are said to conflict with each other if only one of the links can be scheduled reliably on a certain channel simultaneously. (As we will discuss later, conflicts can also occur between a pair of links when those links need to share the same wireless interface). The conflict relation is assumed to be symmetric. The conflict-based interference model is an approximation of the reality – while it does not capture the wireless channel precisely, it is more amenable to analysis, which in turn provides useful insights on performance of wireless networks, as well as insights useful in protocol design. Such conflict-based interference models have been used frequently in the past work as well (e.g., [6]).

Time is assumed to be slotted, with the slot duration being 1 unit time (that is, we use slot duration as the time unit). In each time slot, the scheduler used in the network determines which links should transmit in that time slots, as well as the channel to be used for each such transmission. We now introduce some notation and terminology.

The network is viewed as a collection of directed links, where each link is a pair of nodes capable of direct communication with non-zero rate.

- $\mathcal{L}$ denote the set of directed links in the network.
- $\mathcal{C}$ is the set of all available orthogonal channels. Thus, $|C|$ is the number of available channels.
- We say that a scheduler schedules link-channel pair (l, c) if it schedules link l for transmission on channel c.
- r_l^c denotes rate achievable on link l by operating link l on channel c, provided that no conflicting link is also

scheduled on channel c. We assume that $r_l^c > 0$ for all $l \in \mathcal{L}$ and $c \in \mathcal{C}$ [1]. We also deifne the following terms: $r_{max} = \max_{l \in \mathcal{L}, c \in \mathcal{C}} r_l^c$, and $r_{min} = \min_{l \in \mathcal{L}, c \in \mathcal{C}} r_l^c$ When two conflicting links are scheduled simultaneously on the same channel, both achieve rate 0.

- β_s denotes the "self-skew-ratio", defined as the minimum ratio between rates supportable over *different* channels on a *single* link. Therefore, for any two channels c and d, and any link l, we have $\frac{r_l^d}{r_l^c} \geq \beta_s$. Note that $0 < \beta_s \leq 1$.
- β_c denotes the "cross-skew-ratio", defined as the minimum ratio between rates supportable over the *same* channel on *different* links. Therefore, for any channel c, and any two links l and l', we have that $\frac{r_{l'}^c}{r_l^c} \geq \beta_c$. Note that $0 < \beta_c \leq 1$.

  Let $r_l = \max_{c \in \mathcal{C}} r_l^c$. Let $\sigma_s = \min_{l \in \mathcal{L}} \frac{\sum_{c \in \mathcal{C}} r_l^c}{r_l}$. Note that $\sigma_s \geq 1 + \beta_s(\sigma_s - 1)$. Moreover, typically σ_s will be much larger than this worst-case bound. σ_s is largest when $\beta_s = 1$, and then $\sigma_s = |\mathcal{C}|$.
- $b(l)$ and $e(l)$, respectively, denotes the nodes at the two endpoints of a link. In particular, link l is directed from node $b(l)$ to node $e(l)$.
- $\mathcal{E}(b(l))$and $\mathcal{E}(e(l))$denote the set of links incident on nodes $b(l)$ and $e(l)$, respectively. Thus, the links in $\mathcal{E}(b(l))$and $\mathcal{E}(e(l))$share a node with link l. Since we are focusing on single-interface nodes, this implies that if link l is scheduled in a certain time slot, no other link in $\mathcal{E}(b(l))$or $\mathcal{E}(e(l))$can be scheduled at the same time. This is referred to as an interface conflict. As noted previously, our results (and the notion of interface conflict) can also be extended to the multi-interface case, but the space limitations prevent discussion of this case.
- $\mathbf{I}(l)$ denotes the set of links that conflict with link l when scheduled on the same channel. $\mathbf{I}(l)$ may include links that also have an interface-conflict with link l. By convention, l is considered included in $\mathbf{I}(l)$. Let $A(l) = \mathcal{A}(l)$. Note that $l \in \mathcal{A}(l)$. Links that have an interface conflict with link l are those that belong to $\mathcal{E}(b(l)) \cup \mathcal{E}(e(l)) - \{l\}$; they are also said to be adjacent to link l. The subset of $\mathbf{I}(l)$ comprising interfering links that are not adjacent to l is denoted by $\mathbf{I}'(l)$. Let $I_{max} = \max_l |\mathbf{I}'(l)|$, and let $A_{max} = \max_l |\mathcal{A}(l)|$.
- K denotes the maximum number of non-adjacent links in $\mathbf{I}'(l)$ that can be scheduled on a given channel simultaneously if l is not scheduled on that channel. $K_l(|\mathcal{C}|)$ denotes the maximum number of non-adjacent links in $\mathbf{I}'(l)$ that can be scheduled simultaneously on any of the $|\mathcal{C}|$ channels (without conflicts) if l is not scheduled for transmission. Note that here we exclude links that have an interface conflict with l.
- $K_{|\mathcal{C}|}$ is the largest value of $K_l(|\mathcal{C}|)$ over all links l. That is, $K_{|\mathcal{C}|} = \max_l K_l(|\mathcal{C}|)$. Let $I_{max} = \max_l |\mathbf{I}'(l)|$. It is not hard to see that for single-interface nodes:

$$K \leq K_{|\mathcal{C}|} \leq \min\{K|\mathcal{C}|, I_{max}\} \tag{1}$$

  We remark that the term K as used by us is similar, but not exactly the same as K in [6]. In [6], K denotes the largest number of links that may be scheduled simultaneously if some link l is not scheduled, including links adjacent to l. We exclude the adjacent links. For future reference, we will refer to the quantity defined in [6] as κ instead of K.
- Let γ_l be 0 if there are no other links adjacent to l at either endpoint of l, 1 if there are adjacent links at only one endpoint, and 2 if there are adjacent links at both endpoints.
- γ is the largest value of γ_l over all links l. That is, $\gamma = \max_l \gamma_l$.
- *Load vector*: We consider single-hop traffic flows. That is, each flow originates at one node and ends at an adjacent node, using the link between the two nodes to transmit the traffic (all traffic on a link is clubbed together as one flow). The traffic arrival process for link l is denoted by $\{\lambda(t)\}$.The arrivals in each slot t are

[1]Though we assume that $r_l^c > 0$ for all l, c, the results can be easily generalized to handle the case where $r_l^c = 0$ for some link-channel pairs

i.i.d. with average λ_l. The average load on the network is denoted by *load* vector $\overrightarrow{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_{|\mathcal{L}|}]$, where λ_l denotes the arrival rate for the flow on link l. The load on some links may possibly be 0.

- *Queues*: The packets generated by each flow are first added to a queue maintained at the source node (depending on the algorithm, there could be a single queue for each link, or a queue for each (link, channel) queue).
- *Feasible load vector*: In each time slot, the scheduler used in the network determines which links should transmit and on which channel (recall that each link is a directed link, with a transmitter and a receiver). In different time slots, the scheduler may schedule a different set of links for transmission. A load vector is said to be *feasible*, if there exists a scheduler that can schedule transmissions such that each of the queues in the network remains stable (or, bounded in size) when using that load vector.
- *Link rate vector*: Depending on the schedule chosen in a given slot by the scheduler, each link l will have a certain transmission rate. For instance, using our notation above, if link l is scheduled to transmit on channel c, it will have rate r_l^c (here we assume that, if the scheduler schedules link l on channel c, it does not schedule another conflicting link on that channel). Thus, the *schedule* chosen for a time-slot yields a *link rate vector* for that time slot. Note that *link rate vector* specifies rate of transmission used on each link in a certain time slot. On the other hand, *load vector* specifies the rate at which traffic is generated for each link.
- *Feasible rate region*: The set of all feasible load vectors constitutes the feasible rate-region of the network, and is denoted by Λ. A *throughput-optimal* scheduler is one that is capable of maintaining stable queues for any load vector $\overrightarrow{\lambda} \in \Lambda$.
- *TO-scheduler*: It has been previously shown [1] that a scheduler that maintains a queue q_l for each link l, and then chooses the schedule given by $\text{argmax}_{\overrightarrow{r}} \sum q_l r_l$, where the max is taken over all possible link rate vectors $\overrightarrow{r}$ is throughput-optimal. We will refer to this particular scheduler as *TO-scheduler.* Note that q_l is a function of time, and queue sizes at the start of a time slot are used above for computing the schedule (or link-rate vector) for that slot.
- *Imperfect scheduler*: It is usually difficult to determine the throughput-optimal link-rate allocations above since the problem is typically computationally intractable. Thus, there has been significant recent interest in "imperfect" scheduling policies that can be implemented efficiently. In [2], cross-layer rate-control was studied for an imperfect scheduler that chooses (in each time slot) link-rate vector $\overrightarrow{s}$ such that $\sum q_l s_l \geq \delta \ \text{argmax}_{\overrightarrow{r}} \sum q_l r_l$, for some constant δ ($0 < \delta \leq 1$).

  It was shown [2] that any scheduler with this property can stabilize any load-vector $\overrightarrow{\lambda} \in \delta\Lambda$ – note that if a rate vector $\overrightarrow{\lambda}$ is in Λ, then the rate vector $\delta\overrightarrow{\lambda}$ is in $\delta\Lambda$. $\delta\Lambda$ is also referred to as the δ-reduced rate-region. If a scheduler can stabilize all $\overrightarrow{\lambda} \in \delta\Lambda$, its efficiency-ratio is said to be δ.
- *Maximal scheduler*: Under our interference model, a schedule is said to be maximal if (a) no two links in the schedule conflict with each other, and (b) it is not possible to add any link to the schedule without creating a conflict (either conflict due to interference, or an interface-conflict). The performance of maximal schedulers under various assumptions has been studied in much recent work, e.g., [6], [4], [5], [8]. However, the focus has largely been on single-channel wireless networks. Scheduling in multi-channel networks has been examined in [6], and a queue-loading algorithm has been proposed, using which a maximal scheduler can stabilize any vector in $\frac{1}{\kappa+2}\Lambda$, for arbitrary β_c and β_s values. This paper improve on the prior result, in addition to presenting a new scheduler.

III. Scheduling in Multi-channel Wireless Networks

As has been stated in the previous section, throughput-optimal scheduling is often an intractable problem even in a single-channel network, though imperfect schedulers that achieve a fraction of the stability-region can potentially be implemented in a reasonably efficient manner. When there are multiple channels, but each node has one or few interfaces, an additional degree of complexity is added, in terms of channel coordination. In particular, when the

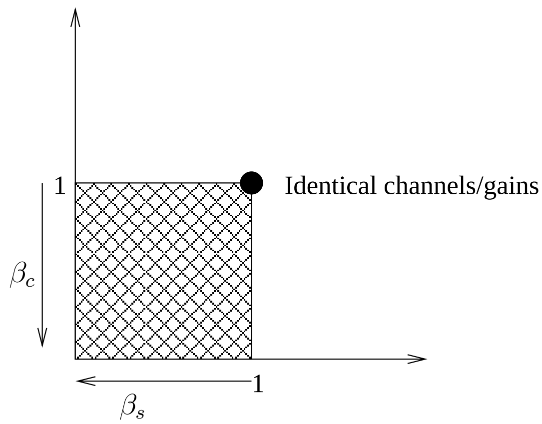

Fig. 1. 2-D visualization of channel heterogeneity

link-channel rates r_l^c can be different for different links l, and channels c, the scheduling complexity is exacerbated by the fact that it is not enough to assign different channels to interfering links; for good performance, the channels must be assigned taking achievable rates into account, i.e., individual channel identities are important. In [6], it was argued that if a simple maximal scheduler is used in such a network, there could possibly be an arbitrary degradation in efficiency-ratio (assuming arbitrary variability in rates) compared to the efficiency-ratio of a maximal scheduler with identical channels. Thus, they proposed a queue-loading rule to control the channels on which a link competes. This rule requires knowledge of of the length of queues at all interfering links, and achieves an efficiency-ratio $\frac{1}{\kappa+2}$.

Variability in channel gains over different links is very much a characteristic of real-world wireless networks, and must indeed be handled by protocols and algorithms. However, if the solutions require extensive information-exchange, the resultant good performance may be offset by the increased overhead. Thus, it is crucial to consider various points of trade-off between information and performance. In this context, the quantities β_s, β_c and σ_s defined in Section II prove to be useful. The quantities β_s and β_c can be viewed as two orthogonal axes for worst-case channel heterogeneity (Fig. 1). The quantity σ_s provides an aggregate (and thus averaged-out) view of heterogeneity along the β_s axis. $\beta_s = 1$ corresponds to a scenario where all channels have identical characteristics, viz., bandwidth, modulation/transmission-rate, etc., and the link-gain is a function solely of the path-loss. $\beta_c = 1$ corresponds to a scenario where all links have the same gain, but the channels may have different characteristics, e.g., an 802.11b channel with a maximum supported data-rate of 11 Mbps, and an 802.11a channel with a maximum supported data-rate of 54 Mbps. In this paper, we show that in a single-interface network, a simple maximal scheduler augmented with local traffic-distribution and threshold rules achieves an efficiency-ratio $\frac{\sigma_s}{K_{|C|}+\max\{1,\gamma\}|C|}$. The noteworthy features of this result are:

1) This scheduler does not require information about queues at interfering links.
2) The performance degradation (compared to the scheduler of [6]) when rates are variable, i.e., $\beta_s, \beta_c \neq 1$ is not arbitrary, and is at worst $\frac{\sigma_s}{|C|} \geq \frac{1+\beta_s(|C|-1)}{|C|} \geq \frac{1}{|C|}$. Thus, even with a purely local information based queue-loading rule, we are able to avoid arbitrary performance degradation even in the worst case. On average, the performance would be much better.
3) In many network scenarios, $\frac{\sigma_s}{K_{|C|}+\max\{1,\gamma\}|C|}$ may actually be better than $\frac{1}{\kappa+2}$. This is particularly likely to happen in networks with single-interface nodes, e.g., suppose we have three channels a, b, c with $r_l^a = 1, r_l^b = 1, r_l^c = 0.5$ for all links l. Then, in the network in Fig. 2 (where the link-interference graph is a star with x radial vertices, and there are no interface-conflicts), we obtain a bound of $\frac{1}{0.4x+1.2}$, whereas the bound of Lin and Rasool is $\frac{1}{x+2}$.

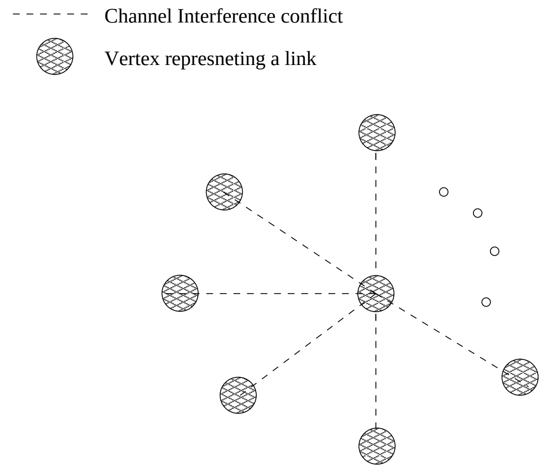

Fig. 2. Example of improved bound on efficiency ratio: link-interference topology is a star with a center link and x radial links

The multi-channel scheduling problem is further complicated if the rates r_l^c are time-varying, i.e., $r_l^c = r_l^c(t)$. However, handling such time-varying rates is beyond the scope of this paper, and we address only the case where rates do not exhibit time-variation.

IV. Summary of Results

For multi-channel wireless networks with single-interface (or single-radio) nodes, we present lower bounds on the efficiency-ratio of a class of maximal schedulers (including both centralized and distributed schedulers), which indicate that the worst-case efficiency-ratio can be higher when there are multiple channels (as compared to the single-channel case). More specifically, we show that:

- The number of links scheduled by any maximal scheduler are within at least a $\max\{\frac{|\mathcal{C}|}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1,K+\gamma\}}\}$ fraction of the maximum number of links activated by any feasible schedule.
- A centralized greedy maximal (CGM) scheduler achieves an efficiency-ratio at least $\max\{\frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1,K+\gamma\}}\}$ This constitutes an improvement over the lower bound for the CGM scheduler proved in [6]. Since $K_{|\mathcal{C}|} \leq K|\mathcal{C}| \leq \kappa|\mathcal{C}|$, this new bound on efficiency-ratio can often be substantially tighter.
- We show that any maximal scheduler, in conjunction with a simple local queue-loading rule, and a threshold-based link-participation rule, achieves an efficiency-ratio of at least $\frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}$. This scheduler is of significant interest as it does not require information about queues at all interfering links.

In the rest of the paper, we elaborate on the results. Most of the proofs are presented in the **Appendix**.

Note that the text below makes the natural assumption that two links that conflict with each other (due to interference or interface-conflict) are **not** scheduled in the same timeslot by any scheduler discussed in the rest of the paper.

V. Maximal Schedulers

We begin the presentation of the results with a result that applies to *all* maximal schedulers.

Theorem 1: Let $\mathcal{S}_{opt}$ denote the set of links scheduled by a scheduler that seeks to maximize the *number* of links scheduled for transmission, and let $\mathcal{S}_{max}$ denote the set of links activated by *any* maximal scheduler. Then the following is true:

$$|\mathcal{S}_{max}| \geq \max\{\frac{|\mathcal{C}|}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1,K+\gamma\}}\}|\mathcal{S}_{opt}| \quad (2)$$

Although we do not use this result directly to prove any of the remaining results, this result makes the interesting point that the availability of multiple channels can potentially improve the ratio of number of scheduled links compared with the optimal scheduler.

VI. Centralized Greedy Maximal Scheduler

A centralized greedy maximal (CGM) scheduler operates as follows in each timeslot: (i) Calculate link weights w_l^c for all links l and channels c. (ii) Sort the link-channel pairs (l,c) in non-increasing order of w_l^c. (iii) Add the first link-channel pair in the list (with highest weight) to the schedule for the timeslot, and remove from the list all link-channel pairs that are no longer feasible (either due to interface or interference conflicts). (iv) Repeat step (iii) until the list is exhausted (thus no more links can be added to the schedule).

In [6], it was shown that this centralized greedy maximal (CGM) scheduler can achieve an approximation-ratio at least $\frac{1}{\kappa+2}$ in a multi-channel network, where κ is the maximum number of links that may possibly be scheduled concurrently as a result of removing another link from the schedule. This bound holds for arbitrary values of β_s and β_c, and variable number of interfaces per node. Though it is tight in that there exists at least one network in which the efficiency-ratio does not exceed the bound, it can be quite loose on average, particularly in networks where there are multiple channels but single-interface nodes. In this section, we prove an improved bound on the efficiency-ratio achievable with the CGM scheduler. Recall that $w_l^c = q_l r_l^c$.

Theorem 2: Let $\mathcal{S}_{opt}$ denote the set of links activated by an "optimal" scheduler that maximizes $\sum w_l^c$ by choosing appropriate link-channel pairs (l,c) for transmission.[2] Let $c^*(l)$ denote the channel assigned to link $l \in \mathcal{S}_{opt}$ by this optimal scheduler. Let $\mathcal{S}_g$ denote the set of links activated by the centralized greedy maximal (CGM) scheduler, and let $c^g(l)$ denote the channel assigned to a link $l \in \mathcal{S}_g$. Then the following is true:

$$\sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} \;\geq\; \max\left\{\frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1,K+\gamma\}}\right\} \sum_{l \in \mathcal{S}_{opt}} w_l^{c^*(l)} \tag{3}$$

The appendix present the proof. The above theorem implies the next result:

Theorem 3: The centralized greedy maximal (CGM) scheduler can stabilize the δ-reduced rate-region, where:

$$\delta = \max\left\{\frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1,K+\gamma\}}\right\}$$

Proof: We earlier discussed a result from [2] that any scheduler, which chooses rate-allocation $\overrightarrow{s}$ such that $\sum q_l s_l \geq \delta$ argmax $\sum q_l r_l$, can stabilize the δ-reduced rate-region. Invoking this result, and Theorem 2, we obtain the above result. ■

Interestingly, the above bound is independent of β_c.

A. Extension to multiple interfaces per-node

We now describe how the result can be extended to networks where each node may have more than one interface.

Given the original network *node-graph* $G=(V,E)$, construct the following transformed graph $G'=(V',E')$:

For each node $v \in V$, if v has m_v interfaces, create m_v nodes $v_1, v_2, ... v_{m_v}$ in V'. For each edge $(u,v) \in E$, where u,v have m_u, m_v interfaces respectively, create edges $(u_i, v_j), 1 \leq i \leq m_u, 1 \leq j \leq m_v$, and set $q_{(u_i,v_j)} = q_{(u,v)}$. Set the achievable channel rate appropriately for each edge in E' and each channel. For example, if channel-rate is solely a function of u,v and c, then: for each channel c, set $r^c_{(u_i,v_j)} = r^c_{(u,v)}$.

The transformed graph G' comprises only single-interface links, and thus Theorem 2 applies to it. Moreover, it is not hard to see that a schedule that maximizes $\sum q_l r_l$ in G' also maximizes $\sum q_l r_l$ in G. Thus the efficiency-ratio from Theorem 2 for network graph G' yields an efficiency-ratio for the performance of the centralized GM scheduler in the multi-interface network.

[2]This optimal scheduler is, in fact, the same as the TO-scheduler discussed earlier, applied to our network model.

Let us briefly touch upon how one would expect the ratio to vary as the number of interfaces at each node increases. Note that the efficiency-ratio depends on $\beta_s, |\mathcal{C}|, K_{|\mathcal{C}|}, \gamma$. Of these β_s and $|\mathcal{C}|$ are always the same for both G and G'. γ is also always the same for any G' derived from a given node-graph G, as it depends only on the number of other node-links incident on either endpoint of a node-link in G (which is a property of the node topology, and not the number of interfaces each node has). However, $K_{|\mathcal{C}|}$ might potentially increase in G' as there are many more non-adjacent interfering *links* when each interface is viewed as a distinct node. Thus, for a given number of channels $|\mathcal{C}|$, one would expect the provable efficiency-ratio to initially decrease as we add more interfaces, and then become static.

While this may initially seem counter-intuitive, this is explained by the observation that multiple orthogonal channels yielded a better efficiency-ratio in the single-interface case since there was more spectral resource, but limited hardware (interfaces) to utilize it. Thus, the additional channels could be effectively used to alleviate the impact of sub-optimal scheduling. When the hardware is commensurate with the number of channels, the situation (compared to an optimal scheduler) increasingly starts to resemble a single-channel single-interface network.

B. The special case of $|\mathcal{C}|$ interfaces per node

Let us consider the special case where each node in the network has $|\mathcal{C}|$ interfaces, and achievable rate on a link between nodes u, v and all channels $c \in \mathcal{C}$ is solely a function of u, v and c (and not of the interfaces used). In this case, it is possible to obtain a simpler transformation. Given the original network node-graph $G = (V, E)$, construct $|\mathcal{C}|$ copies of this graph, viz., $G_1, G_2, ..., G_{|\mathcal{C}|}$, and view each node in each graph as having a single-interface, and each network having access to a single channel. Then each network graph G_i can be viewed in isolation, and the throughput obtained in the original graph is the sum of the throughputs in each graph. From Theorem 2, in each graph we can show that the CGM scheduler is within $\max\{1, \frac{1}{K+\gamma}\}$ of the optimal. Thus, even in the overall network, the CGM scheduler is within $\max\{1, \frac{1}{K+\gamma}\}$ of the optimal.

VII. A Simple Maximal Scheduler with Thresholds

In this section we present a simple extension to multiple channels of the result of [4] for a maximal scheduler with threshold-based participation. This serves as a precursor for the results of the next section.

The set of all links in denoted by $\mathcal{L}$. The arrival process of each link l is denoted by $\{\lambda_l(t)\}$. For a given link l, the arrivals $\lambda_l(t)$ are i.i.d., and $E[\lambda_l(t)] = \lambda_l$. However, we make no assumptions about independence of arrival processes of two different links. Moreover, $E[\lambda_l(t)\lambda_k(t)]$ is bounded, i.e., $E[\lambda_l(t)\lambda_k(t)] \leq \eta$ for all $l, k \in \mathcal{L}$, where η is a suitable constant.

Theorem 4: If $\beta_s = 1$, i.e., $r_l^c = r_l$ for all $l \in \mathcal{L}$, then the following scheduling policy stabilizes the network whenever $\sum\limits_{k \in \mathcal{A}(l)} \frac{\lambda_k}{r_k} + \frac{1}{|\mathcal{C}|} \sum\limits_{k \in \mathbf{I}'(l)} \frac{\lambda_k}{r_k} < 1$, for all $l \in \mathcal{L}$:
In time-slot t, only links l with $q_l(t) \geq r_l$ participate, and a maximal schedule is computed.
The proof is presented in the Appendix.

VIII. A Rate-Proportional Maximal Multi-Channel (RPMMC) Scheduler

The set of all links in denoted by $\mathcal{L}$. The arrival process for link l is i.i.d. over all time-slots t, and is denoted by $\{\lambda_l(t)\}$, with $E[\lambda_l(t)] = \lambda_l$. We make no assumption about independence of arrival processes for two links l, k. However, we consider only the class of arrival processes for which $E[\lambda_l(t)\lambda_k(t)]$ is bounded, i.e., $E[\lambda_l(t)\lambda_k(t)] \leq \eta$ for all $l, k \in \mathcal{L}$, where η is a suitable constant.

Consider the following scheduler:

Rate-Proportional Maximal Multi-Channel (RPMMC) Scheduler

Each link maintains a queue for each channel. The length of the queue for link l and channel c at time t is denoted by $q_l^c(t)$. In time-slot t: only those link-channel pairs with $q_l^c(t) \geq r_l^c$ participate, and the scheduler computes a maximal schedule. The new arrivals during this slot, i.e. $\lambda_l(t)$ are assigned to channel-queues in proportion to the rates, i.e., $\lambda_l^c(t) = \frac{\lambda_l r_l^c}{\sum_{b \in C} r_l^b}$

Theorem 5: The RPMMC scheduler stabilizes the network for any load-vector within the $\frac{\sigma_s}{K_{|C|}+\max\{1,\gamma\}|C|}$-reduced rate-region.

The proof is presented in the Appendix.

Corollary 1: When $\beta_s = 1$, the RPMMC scheduler achieves an efficiency ratio of $\frac{|C|}{K_{|C|}+\max\{1,\gamma\}|C|}$.

IX. Conclusion

We have presented bounds on the efficiency-ratio achieved by certain maximal multi-channel schedulers. In particular, we have proposed a scheduler that can achieve acceptable performance with limited information. Promising directions for future research include designing low-overhead algorithms for computing maximal schedules in multi-channel networks, and further exploring the trade-off between information-exchange and performance.

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Appendix

Recall that $\mathcal{S}_{max}$ is defined in the statement of Theorem 1 as the set of links scheduled by any maximal scheduler. Also recall that $\mathcal{S}_g$ is defined in the statement of Theorem 2 as the set of links scheduled by the CGM scheduler. Also, $\mathcal{S}_{opt}$ is defined as the set of links scheduled by the optimal scheduler, where the notion of optimality is as defined in each of Theorem 1 and Theorem 2. In the following proofs, we may use the term $\mathcal{S}_{both}$, which is a concise way to refer to either $\mathcal{S}_g \cap \mathcal{S}_{opt}$ or $\mathcal{S}_{max} \cap \mathcal{S}_{opt}$, depending on the context of that particular proof.

Proof: (*Proof of Theorem 1*) Consider $l \in \mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}$. Denote by $c^m(l)$ the channel on which l is scheduled in $\mathcal{S}_{max}$. Since l was not scheduled by the maximal scheduler, this implies that at least one of the following events must be true:

1) Condition 1: $\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}} \cap (\mathcal{E}(b(l)) \cup \mathcal{E}(e(l))) \neq \phi$.
2) Condition 2: For each channel $c \in \mathcal{C}$, there exists some link $l'_c \in \mathcal{S}_{max} \cap \mathbf{I}'(l)$, such that $c^m(l'_c) = c$.

Now, define sets A_{if} and A_{in} as follows:

$\mathcal{A}_{if} = \{l : l \in \mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}$ and Condition 1 holds$\}$. $\mathcal{A}_{in} = (\mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}) - \mathcal{A}_{if}$

Thus $\mathcal{A}_{if}$ comprises the set of links in $\mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}$ that are blocked in the maximal-schedule by interface-conflicts, while $\mathcal{A}_{in}$ comprises the set of links in $\mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}$ that are blocked in the maximal-schedule purely by channel-interference conflicts. For each $l \in \mathcal{A}_{in}$, let $\mathcal{Y}_l = \bigcup_{c \in C} \{l'_c : l'_c \in \mathcal{S}_{max} \cap \mathbf{I}'(l), c^m(l'_c) = c\}$. Any link $l' \in \mathcal{S}_{max}$ can occur in the $\mathcal{Y}_l$ of at most $K_{|\mathcal{C}|}$ non-adjacent links $l \in \mathcal{S}_{opt}$. Thus, it follows that:

$$|\mathcal{C}||\mathcal{A}_{in}| \leq K_{|\mathcal{C}|}|\mathcal{S}_{max}| \tag{4}$$

Any interface-conflicts experienced by links in $\mathcal{S}_{opt} \cap \overline{\mathcal{S}_{max}}$ must necessarily be caused by links in $\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}$. Since a link can only cause up to γ interface-conflicts, we obtain that:

$$|\mathcal{A}_{if}| \leq \gamma\, |\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}| \tag{5}$$

Thus we obtain the following:

$$\begin{aligned}
\frac{|\mathcal{S}_{opt}|}{|\mathcal{S}_{max}|} &= \frac{|\mathcal{S}_{max} \cap \mathcal{S}_{opt}| + |\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}|}{|\mathcal{S}_{max}|} = \frac{|\mathcal{S}_{max} \cap \mathcal{S}_{opt}| + |\mathcal{A}_{if}| + |\mathcal{A}_{in}|}{|\mathcal{S}_{max}|} \\
&\leq \frac{|\mathcal{S}_{max} \cap \mathcal{S}_{opt}| + \gamma|\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}| + \frac{K_{|\mathcal{C}|}}{|\mathcal{C}|}|\mathcal{S}_{max}|}{|\mathcal{S}_{max}|} \text{ from (5), (4)} \\
&= \frac{|\mathcal{S}_{max} \cap \mathcal{S}_{opt}| + |\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}| + (\gamma - 1)|\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}| + \frac{K_{|\mathcal{C}|}}{c}|\mathcal{S}_{max}|}{|\mathcal{S}_{max}|} \\
&= \frac{|\mathcal{S}_{max}| + (\gamma - 1)|\mathcal{S}_{max} \cap \overline{\mathcal{S}_{opt}}| + \frac{K_{|\mathcal{C}|}}{|\mathcal{C}|}|\mathcal{S}_{max}|}{|\mathcal{S}_{max}|} \leq \frac{|\mathcal{S}_{max}| + \max\{0, \gamma - 1\}|\mathcal{S}_{max}| + \frac{K_{|\mathcal{C}|}}{|\mathcal{C}|}|\mathcal{S}_{max}|}{|\mathcal{S}_{max}|} \\
&= 1 + \max\{0, \gamma - 1\} + \frac{K_{|\mathcal{C}|}}{|\mathcal{C}|} = \max\{1, \gamma\} + \frac{K_{|\mathcal{C}|}}{|\mathcal{C}|}
\end{aligned} \tag{6}$$

We now prove another bound, viz. $|\mathcal{S}_{max}| \geq \frac{1}{K + \max\{1,\gamma\}}|\mathcal{S}_{opt}|$.

Consider a link l that is scheduled on some channel c in $\mathcal{S}_{max}$. Either l is scheduled on channel c even in $\mathcal{S}_{opt}$, or if l is not scheduled in $\mathcal{S}_{opt}$, at most K links in $\mathbf{I}'(l)$, and γ links in $\mathcal{A}(l) - \{l\}$ could have been scheduled on channel c in $\mathcal{S}_{opt}$. Thus:

$$\frac{|\mathcal{S}_{opt}|}{|\mathcal{S}_{max}|} \leq \max\{1, K + \gamma\} \tag{7}$$

Thus: $|\mathcal{S}_{max}| \geq \max\{\frac{|\mathcal{C}|}{K_{|\mathcal{C}|} + \max\{1,\gamma\}|\mathcal{C}|}, \frac{1}{\max\{1, K+\gamma\}}\}|\mathcal{S}_{opt}|$. ■

Proof: (*Proof of Theorem 2*)

Denote by $c^*(l)$ the channel on which $l \in \mathcal{S}_{opt}$ is activated by the optimal scheduler. $c^g(l)$ is the channel on which $l \in \mathcal{S}_g$ is activated by the CGM scheduler.

Consider $l \in \mathcal{S}_{opt} \cap \overline{\mathcal{S}_g}$. Since l was not scheduled by the CGM scheduler, this implies that at least one of the following two conditions must be true:

1) Condition 1: There exists a link $l' \in \mathcal{S}_g \cap \overline{\mathcal{S}_{opt}} \cap (\mathcal{E}(b(l)) \cup \mathcal{E}(e(l)))$ such that $w_{l'}^{c^g(l')} \geq w_l^c$ for at least one channel $c \in \mathcal{C}$.

2) Condition 2: For each channel $c \in \mathcal{C}$, there exists some link $l'_c \in \mathcal{S}_g \cap \mathbf{I}'(l)$ such that $w^c_{l'_c} \geq w^c_l$.

Now, define sets A_{if} and A_{in} as follows:

$\mathcal{A}_{if} = \{l : l \in \mathcal{S}_{opt} \cap \overline{\mathcal{S}_g} \quad \text{and Condition 1 holds}\}$.

$\mathcal{A}_{in} = (\mathcal{S}_{opt} \cap \overline{\mathcal{S}_g}) - \mathcal{A}_{if}$

Let $\mathcal{S}_{both,more} = \{l : l \in \mathcal{S}_g \cap \mathcal{S}_{opt}, w_l^{c^g(l)} \geq w_l^{c^*(l)}\}$

Let $\mathcal{S}_{both,less} = \{l : l \in \mathcal{S}_g \cap \mathcal{S}_{opt}, w_l^{c^g(l)} < w_l^{c^*(l)}\}$

Then $\mathcal{S}_{both,more}$ and $\mathcal{S}_{both,less}$ constitute a partition of $\mathcal{S}_g \cap \mathcal{S}_{opt}$.

Let $\mathcal{A}_{if,1} = \{l : l \in \mathcal{A}_{if}, c^*(l)$ was not available to l when l's first interface got used up during CGM scheduling$\}$

Let $\mathcal{A}_{if,2} = \{l : l \in \mathcal{A}_{if}, c^*(l)$ was still available to l when l's first interface got used up during CGM scheduling$\}$

From the greedy nature of the scheduler, if a link $l' \in \mathbf{I}'(l)$ was scheduled on some $c \in \mathcal{C}$ in $\mathcal{S}_g$ while l was still schedulable on some subset of channels $\mathcal{D}_l \subseteq \mathcal{C}$, this implies that $w^c_{l'} \geq w^d_l$ for all $d \in \mathcal{D}_l$.

Note that for all $l \in \mathcal{A}_{if,1}$, and $\mathcal{S}_{both,less}$, it must be true that some link $l' \in \mathbf{I}'(l)$ was assigned $c^*(l)$ in $\mathcal{S}_g$ while l was still schedulable on $c^*(l)$, i.e., $c^*(l) \in \mathcal{D}_l$, where $\mathcal{D}_l$ is the set of channels on which l was still schedulable when $c^*(l)$ was first assigned to some link in $\mathbf{I}'(l)$.

Moreover, it is true that at the time when $l \in \mathcal{S}_{both,less}$ was assigned $c^g(l)$, all other $c \in \mathcal{C}$ with $r^c_l > r_l^{c^g(l)}$ were already assigned to some other $l' \in \mathbf{I}'(l)$, with $w_{l'}^{c^g(l')} = w^c_{l'} \geq w^c_l$. Thus, for all $d \in \mathcal{D}_l, r^d_l \leq r_l^{c^g(l)}$, and $|\mathcal{D}_l| \leq |\mathcal{C}|-1$ since $c^*(l) \notin \mathcal{D}_l$. Therefore for each $l \in \mathcal{S}_{both,less}$: $\sum_{c \in \mathcal{C}-\mathcal{D}_l} \sum_{\substack{l' \in \mathbf{I}'(l) \\ c^g(l')=c}} w_{l'}^{c^g(l')} \geq \sum_{c \in \mathcal{C}} w^c_l - \sum_{d \in \mathcal{D}_l} w^d_l \geq \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1) w_l^{c^g(l)}$.

Thus: $\sum_{l \in \mathcal{S}_{both,less}} \left(\sum_{c \in \mathcal{C}-\mathcal{D}_l} \sum_{\substack{l' \in \mathbf{I}'(l) \\ c^g(l')=c}} w_{l'}^{c^g(l')} \right) \geq \sum_{l \in \mathcal{S}_{both,less}} \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1) \sum_{l \in \mathcal{S}_{both,less}} w_l^{c^g(l)}$.

Similarly, it is true that if $l' \in \mathcal{A}(l) \cap (\mathcal{S}_g \cap \overline{\mathcal{S}_{opt}})$ was assigned a channel $c^g(l')$ in $\mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}$ while $l \in \mathcal{A}_{if,1}$ was still schedulable on some subset of channels $\mathcal{D}_l \subseteq \mathcal{C} - \{c^*(l)\}$ then $w_{l'}^{c^g(l')} \geq w^d_l$ for all $d \in \mathcal{D}_l$, and $|\mathcal{D}_l| \leq |\mathcal{C}|-1$ since $c^*(l) \notin \mathcal{D}_l$. Let us denote by $f(l)$ the link l' in $\mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}$ that is the cause of blocking the first interface of link $l \in \mathcal{A}_{if}$.

Let $B = \sum_{l \in \mathcal{A}_{if,1}} w_{f(l)}^{c^g(f(l))}$. Then, it is evident that $B \leq \gamma \sum_{l \in \mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}} w_l^{c^g(l)}$. Furthermore, $\sum_{c \in \mathcal{C}-\mathcal{D}_l} \sum_{\substack{l' \in \mathbf{I}'(l) \\ c^g(l')=c}} w_{l'}^{c^g(l')} \geq \sum_{c \in \mathcal{C}} w^c_l - \sum_{d \in \mathcal{D}_l} w^d_l \geq \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1) w_{f(l)}^{c^g(f(l))}$, and resultantly $\sum_{l \in \mathcal{A}_{if,1}} \left(\sum_{c \in \mathcal{C}-\mathcal{D}_l} \sum_{\substack{l' \in \mathbf{I}'(l) \\ c^g(l')=c}} w_{l'}^{c^g(l')} \right) \geq \sum_{l \in \mathcal{A}_{if,1}} \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1)B$.

In light of this, and the definition of A_{in} and σ_s:

$$\begin{aligned}
&\sum_{l \in \mathcal{S}_{both,less}} \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1) \sum_{l \in \mathcal{S}_{both,less}} w_l^{c^g(l)} + \sum_{l \in \mathcal{A}_{if,1}} \sum_{c \in \mathcal{C}} w^c_l - (|\mathcal{C}|-1)B + \sum_{l \in \mathcal{A}_{in}} \sum_{c \in \mathcal{C}} w^c_l \leq K_{|\mathcal{C}|} \sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} \\
&\therefore \sigma_s \Big(\sum_{l \in \mathcal{S}_{both,less}} w_l^{c^*(l)} + \sum_{l \in \mathcal{A}_{if,1}} w_l^{c^*(l)} + \sum_{l \in \mathcal{A}_{in}} w_l^{c^*(l)} \Big) \leq K_{|\mathcal{C}|} \sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} + (|\mathcal{C}|-1) \Big(\sum_{l \in \mathcal{S}_{both,less}} w_l^{c^g(l)} + B \Big) \\
&\therefore \sum_{l \in \mathcal{S}_{both,less}} w_l^{c^*(l)} + \sum_{l \in \mathcal{A}_{if,1}} w_l^{c^*(l)} + \sum_{l \in \mathcal{A}_{in}} w_l^{c^*(l)} \leq \frac{K_{|\mathcal{C}|}}{\sigma_s} \sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} + \frac{|\mathcal{C}|-1}{\sigma_s} \Big(\sum_{l \in \mathcal{S}_{both,less}} w_l^{c^g(l)} + B \Big)
\end{aligned} \tag{8}$$

Furthermore, if a link l' adjacent to l was scheduled in $\mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}$ at a time when l was still schedulable on $c^*(l)$, as is the case for links in $\mathcal{A}_{if,2}$, then it implies that $w_{l'}^{c^g(l')} \geq w_l^{c^*(l)}$. Let $E = \sum_{l \in \mathcal{A}_{if,2}} w_{f(l)}^{c^g(f(l))}$. Thus we obtain:

$$\begin{aligned}
B + \sum_{l \in \mathcal{A}_{if,2}} w_l^{c^*(l)} &\leq B + E \leq \gamma \sum_{l \in \mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} \\
\therefore \sum_{l \in \mathcal{A}_{if,2}} w_l^{c^*(l)} &\leq \gamma \sum_{l \in \mathcal{S}_g \cap \overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B
\end{aligned} \tag{9}$$

This yields the following:

$$\frac{\sum_{l\in\mathcal{S}_{opt}} w_l^{c^*(l)}}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} = \frac{\sum_{l\in\mathcal{S}_{both}} w_l^{c^*(l)} + \sum_{l\in\mathcal{S}_{opt}\cap\overline{\mathcal{S}_g}} w_l^{c^*(l)}}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} = \frac{\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^*(l)} + \sum_{l\in\mathcal{S}_{both,less}} w_l^{c^*(l)} + \sum_{l\in\mathcal{A}_{if,1}} w_l^{c^*(l)} + \sum_{l\in\mathcal{A}_{if,2}} w_l^{c^*(l)} + \sum_{l\in\mathcal{A}_{in}} w_l^{c^*(l)}}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}}$$

$$= \frac{\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^*(l)} + (\sum_{l\in\mathcal{S}_{both,less}} w_l^{c^*(l)} + \sum_{l\in\mathcal{A}_{if,1}} w_l^{c^*(l)} + \sum_{l\in\mathcal{A}_{in}} w_l^{c^*(l)}) + \sum_{l\in\mathcal{A}_{if,2}} w_l^{c^*(l)}}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}}$$

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \frac{K_{|\mathcal{C}|}\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + (|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_{both,less}} w_l^{c^g(l)} + B)}{\sigma_s} + \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B \right)$$

from (8), (9)

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \frac{K_{|\mathcal{C}|}\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + (|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_{both,less}} w_l^{c^g(l)} + B)}{\sigma_s} + \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B \right)$$

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \frac{K_{|\mathcal{C}|}\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + (|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} + B)}{\sigma_s} + \frac{\sigma_s(\gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B)}{\sigma_s} \right)$$

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\frac{|\mathcal{C}|}{\sigma_s} \sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \frac{K_{|\mathcal{C}|}\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + (|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} + B)}{\sigma_s} + \frac{|\mathcal{C}|\gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - (|\mathcal{C}|-1)B}{\sigma_s} \right) \text{ noting that } \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B \geq 0$$

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\frac{(|\mathcal{C}|-1)\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + K_{|\mathcal{C}|}\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}}{\sigma_s} + \frac{(|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} - \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} + B + \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)} - B) + \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)}}{\sigma_s} \right)$$

$$\leq \frac{1}{\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)}} \left(\frac{K_{|\mathcal{C}|}}{\sigma_s} \sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + \frac{(|\mathcal{C}|-1)(\sum_{l\in\mathcal{S}_g} w_l^{c^g(l)} + (\gamma-1)\sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)}) + (\sum_{l\in\mathcal{S}_{both,more}} w_l^{c^g(l)} + \gamma \sum_{l\in\mathcal{S}_g\cap\overline{\mathcal{S}_{opt}}} w_l^{c^g(l)})}{\sigma_s} \right)$$

$$\leq \frac{K_{|\mathcal{C}|} + (|\mathcal{C}|-1)(1+\max\{0,\gamma-1\}) + \max\{1,\gamma\}}{\sigma_s}$$

$$= \frac{K_{|\mathcal{C}|} + \max\{1,\gamma\}|\mathcal{C}|}{\sigma_s} \tag{10}$$

Thus $\sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} \geq \frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|} \sum_{l \in \mathcal{S}_{opt}} w_l^{c^*(l)}$. When $\beta_s = 1$, this reduces to a ratio of $\frac{|\mathcal{C}|}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}$.

We now prove another bound by showing that:

$$\sum_{l \in \mathcal{S}_g} w_l^{c^g(l)} \geq \frac{1}{\max\{1, K+\gamma\}} \sum_{l \in \mathcal{S}_{opt}} w_l^{c^*(l)} \tag{11}$$

This is obtained via an argument very similar to that used in [6] to prove a bound of $\frac{1}{\kappa+2}$ for the CGM scheduler, except that we refine the analysis based on a more precise characterization of the interference topology:

Consider any link l in $\mathcal{S}_g$. Either l is scheduled on $c^g(l)$ even in $\mathcal{S}_{opt}$, or if l is not scheduled on $c^g(l)$, at most K links in $\mathbf{I}'(l)$, and γ links in $\mathcal{A}(l) - \{l\}$ could have been scheduled on $c^g(l)$ in $\mathcal{S}_{opt}$, and each would have weight less than or equal to $w_l^{c^g(l)}$. Thus:

$$\frac{\sum_{l \in \mathcal{S}_{opt}} w_l^{c^*(l)}}{\sum_{l \in \mathcal{S}_g} w_l^{c^g(l)}} \leq \max\{1, K+\gamma\} \tag{12}$$

■

Proof: (*Proof of Theorem 4*) We describe a proof of stability based on Lyapunov drift analysis.

We adopt the following convention: at the beginning of each time-slot, the scheduling decisions are taken, and transmissions occur. Then new arrivals occur at the end of the slot (thus new arrivals cannot be transmitted in the same slot, even if there is spare bandwidth).

Let the queue-length of link l at slot t be denoted by $q_l(t)$. Let the rate-allocated to link l in slot t over channel c be denoted by $x_l^c(t)$. Since we are considering single-interface nodes, and $\beta_s = 1$, and a link only participates in a slot if $q_l(t) \geq r_l$, it follows that $\sum_{c \in \mathcal{C}} x_l^c(t) \in \{0, r_l\}$ and at most one of the $x_l^c(t)$'s is non-zero for a link l.

We assume that $r_l > 0$ for all $l \in \mathcal{L}$, since any feasible load-vector must have $\lambda_l = 0$ for any link l with $r_l = 0$, and thus such links can be ignored/eliminated from consideration.

The following is trivially true for any feasible set of arrival processes:

$$\lambda_l \leq r_l \ \forall l \in \mathcal{L} \tag{13}$$

The queue dynamics are as follows:

$$q_l(t+1) = q_l(t) + \lambda_l - \sum_{c \in \mathcal{C}} x_l^c(t) \tag{14}$$

We define the following Lyapunov function:

$$V_q(\overrightarrow{q}) = \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k \in \mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k \in \mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \tag{15}$$

This Lyapunov function is similar in form to that used in [4].

Then, it can be seen that:

$$
\begin{aligned}
V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) &= \sum_{l\in\mathcal{L}} \frac{q_l(t+1)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t+1)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t+1)}{r_k} \right) \\
&\quad - \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \\
&= \sum_{l\in\mathcal{L}} \frac{(q_l(t)+q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t)+q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t)+q_k(t+1)-q_k(t))}{r_k} \right) \\
&\quad - \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \\
&= \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) + \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right) \\
&\quad + \sum_{l\in\mathcal{L}} \frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \\
&+ \sum_{l\in\mathcal{L}} \frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right) - \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \\
&= \sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right) + \sum_{l\in\mathcal{L}} \frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{q_k(t)}{r_k} \right) \\
&+ \sum_{l\in\mathcal{L}} \frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right) \\
&= 2\sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right) \\
&+ \sum_{l\in\mathcal{L}} \frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} + \frac{1}{|\mathcal{C}|} \sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right)
\end{aligned}
\tag{16}
$$

Denote by $\mathcal{L}'(t)$ the set of links l for which $q_l(t) \geq r_l$ and which therefore participate in the scheduling process during slot t. Since the computed schedule is always maximal, it follows that for each $l \in \mathcal{L}'(t)$, either (1) l is activated in slot t, or (2) some link $k \in (\mathcal{A}(l) - \{l\}) \cap \mathcal{L}'(t)$ (i.e., adjacent to l) is activated on some channel, thereby blocking l through interface-conflict, or (3) for each channel $c \in \mathcal{C}$, at least one link $k_c \in \mathcal{L}'(t) \cap \mathbf{I}'(l)$ is activated in slot t on channel c, thereby blocking l through interference-conflict.

$$
\begin{aligned}
&E[V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) | \overrightarrow{q}(t)] \\
&\leq 2\sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(E[\sum_{k\in\mathcal{A}(l)} \frac{q_k(t+1)-q_k(t)}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{q_k(t+1)-q_k(t)}{r_k} \right) \\
&+ \sum_{l\in\mathcal{L}} E\left[\frac{(q_l(t+1)-q_l(t))}{r_l} \left(\sum_{k\in\mathcal{A}(l)} E[\frac{q_k(t+1)-q_k(t)}{r_k}] + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{(q_k(t+1)-q_k(t))}{r_k} \right)\right] \\
&\leq 2\sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} E\left[\frac{\lambda_k(t) - \sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} \right] + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} E\left[\frac{\lambda_k(t) - \sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} \right] \right) \\
&\qquad\qquad + \sum_{l\in\mathcal{L}} E\left[\frac{\lambda_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{\lambda_k(t)}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\lambda_k(t)}{r_k} \right) \right] \\
&\leq 2\sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{\lambda_k}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\lambda_k}{r_k} - E\left[\sum_{k\in\mathcal{A}(l)} \frac{\sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} \right] \right) + C_1 \\
&\leq 2\sum_{l\in\mathcal{L}'(t)} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{\lambda_k}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\lambda_k}{r_k} - E\left[\sum_{k\in\mathcal{A}(l)} \frac{\sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\sum_{c\in\mathcal{C}} x_k^c(t)}{r_k} \right] \right) \\
&\qquad\qquad + 2\sum_{l\in\mathcal{L}-\mathcal{L}'(t)} \frac{q_l(t)}{r_l} \left(\sum_{k\in\mathcal{A}(l)} \frac{\lambda_k}{r_k} + \frac{1}{|\mathcal{C}|}\sum_{k\in\mathbf{I}'(l)} \frac{\lambda_k}{r_k} \right) + C_1 \\
&\leq 2\sum_{l\in\mathcal{L}'(t)} \frac{q_l(t)}{r_l}(1-\varepsilon-1) - 2\sum_{l\in\mathcal{L}-\mathcal{L}'(t)} \frac{q_l(t)}{r_l}\varepsilon + 2\sum_{l\in\mathcal{L}-\mathcal{L}'(t)} \frac{q_l(t)}{r_l}\varepsilon + 2\sum_{l\in\mathcal{L}-\mathcal{L}'(t)} \frac{q_l(t)}{r_l}(\sum_{k\in\mathcal{A}(l)} \frac{\lambda_k}{r_k} + \sum_{k\in\mathbf{I}'(l)} \frac{\lambda_k}{r_k}) + C_1 \\
&\qquad\qquad\qquad\qquad \text{(substracting and adding back } 2\sum_{l\in\mathcal{L}-\mathcal{L}'(t)} \frac{q_l(t)}{r_l}\varepsilon) \\
&\leq 2\sum_{l\in\mathcal{L}} \frac{q_l(t)}{r_l}(1-\varepsilon-1) + C_2 \\
&= \frac{-2\varepsilon}{r_{max}} \sum_{l\in\mathcal{L}} q_l(t) + C_2
\end{aligned}
\tag{17}
$$

where $C_1 = \frac{2|\mathcal{L}|\eta(A_{max}+\frac{I_{max}}{|\mathcal{C}|})}{r_{min}}$, and $C_2 = C_1 + 2\varepsilon|\mathcal{L}| + 2|\mathcal{L}|(A_{max} + \frac{I_{max}}{|\mathcal{C}|})$.

Invoking Lemma 2 from [9], this proves stability. ■

Proof: (Proof of Theorem 5) We describe a proof of stability based on Lyapunov drift analysis.

We adopt the following convention: at the beginning of each time-slot, the scheduling decisions are taken, and transmissions occur. Then new arrivals occur at the end of the slot (thus new arrivals cannot be transmitted in the same slot, even if there is spare bandwidth).

Let the queue-length of the queue for link l and channel c at the start of time-slot t be denoted by $q_l^c(t)$. Let the rate-allocated to link l in slot t over channel c be denoted by $x_l^c(t)$. Since we are considering single-interface nodes, at most one of the $x_l^c(t)$'s is non-zero for a link l. Furthermore $x_l^c(t) = 0$ if link l is not scheduled over channel c in slot t, and $x_l^c(t) = r_l^c$ else.

Recall that $r_l = \max_{c\in\mathcal{C}} r_l^c$. By assumption $r_l^c > 0$ for all $l \in \mathcal{L}, c \in \mathcal{C}$. However, as noted earlier, the result can be easily generalized to allow some of these to be 0.

The queue dynamics are as follows:

$$q_l^c(t+1) = q_l^c(t) + \lambda_l^c(t) - x_l^c(t) \text{ where } \lambda_l^c(t) = \frac{\lambda_l(t) r_l^c}{\sum\limits_{b\in\mathcal{C}} r_l^b} \tag{18}$$

We define the following Lyapunov function:

$$V_q(\overrightarrow{q}) = \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \right] \tag{19}$$

This Lyapunov function is somewhat similar in form to that used in [4], but now uses per-channel queue-lengths. Then, it can be seen that:

$$\begin{aligned}
V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) &= \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t+1)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t+1)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t+1)}{r_k^c} \right) \right] \\
&\quad - \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \right] \\
&= \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t) + q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t) + q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t) + q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&\quad - \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \right] \\
&= \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \frac{q_l^c(t)}{r_l^c} \left[\left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&\quad + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \right] \\
&\quad + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&\quad - \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \\
&= \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&\quad + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{q_k^c(t)}{r_k^c} \right) \right] \\
&\quad + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&= 2 \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{q_l^c(t)}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right] \\
&\quad + \sum_{l\in\mathcal{L}} \sum_{c\in\mathcal{C}} \left[\frac{(q_l^c(t+1) - q_l^c(t))}{r_l^c} \left(\sum_{k\in\mathcal{A}(l)} \sum_{d\in\mathcal{C}} \frac{(q_k^d(t+1) - q_k^d(t))}{r_k^d} + \sum_{k\in\mathbf{I}'(l)} \frac{(q_k^c(t+1) - q_k^c(t))}{r_k^c} \right) \right]
\end{aligned} \tag{20}$$

Denote by $\mathcal{L}'(t)$ the set of link-channel pairs (l,c) for which $q_l^c(t) \geq r_l^c$. This set of link-channel pairs participates in the scheduling process for slot t. By assumption, the scheduler computes a maximal schedule over all participating links. Thus, for all $l \in \mathcal{L}$ and $c \in \mathcal{C}$, whenever $q_l^c(t) \geq r_l^c$:

$$\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{x_k^d(t)}{r_k^d} + \sum_{k \in \mathbf{I}'(l)} \frac{x_k^c(t)}{r_k^c} \geq 1 \tag{21}$$

If $\overrightarrow{\lambda}$ lies within the $\frac{\sigma_s}{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}$-reduced rate-region, then, by assumption, there exists some scheduling algorithm that achieves stability with load vector $(\frac{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}{\sigma_s})\overrightarrow{\lambda}$. Similar to [6], we can argue that this implies existence of an average service-rate vector $\widetilde{x_l^c}$ for all l, c satisfying the following, for some $\varepsilon > 0$:

$$(1+\varepsilon)^2 \left(\frac{K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|}{\sigma_s}\right)\lambda_l \leq \sum_{c \in \mathcal{C}} \widetilde{x_l^c} \text{ for all links } l \tag{22}$$

$$\sum_{k \in \mathbf{I}'(l)} \sum_{c \in \mathcal{C}} \frac{\widetilde{x_k^c}}{r_k^c} \leq K_{|\mathcal{C}|} \text{ for all links } l \tag{23}$$

$$\sum_{k \in \mathcal{A}(l)} \sum_{c \in \mathcal{C}} \frac{\widetilde{x_k^c}}{r_k^c} \leq \max\{1,\gamma\} \text{ for all links } l \tag{24}$$

Set $\overline{x_l^c} = \frac{\widetilde{x_l^c}\sigma_s}{(1+\varepsilon)(K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|)}$. Then from (22), (23) and (24), we obtain that:

$$(1+\varepsilon)\lambda_l \leq \sum_{c \in \mathcal{C}} \overline{x_l^c} \text{ for all links } l \tag{25}$$

$$\sum_{c \in \mathcal{C}} \sum_{k \in \mathbf{I}'(l)} \frac{\overline{x_k^c}}{r_k^c} \leq \frac{K_{|\mathcal{C}|}\sigma_s}{(1+\varepsilon)(K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|)} \text{ for all links } l \tag{26}$$

$$\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{\overline{x_k^d}}{r_k^d} \leq \frac{\max\{1,\gamma\}\sigma_s}{(1+\varepsilon)(K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|)} \text{ for all links } l \tag{27}$$

This yields that for all links l:

$$\begin{aligned} \sum_{b \in \mathcal{C}} \left(\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{\overline{x_k^d}}{r_k^d} + \sum_{k \in \mathbf{I}'(l)} \frac{\overline{x_k^b}}{r_k^b} \right) &= \left(|\mathcal{C}| \sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{\overline{x_k^d}}{r_k^d} + \sum_{k \in \mathbf{I}'(l)} \sum_{b \in \mathcal{C}} \frac{\overline{x_k^b}}{r_k^b} \right) \\ \leq \frac{\max\{1,\gamma\}\sigma_s|\mathcal{C}|}{(1+\varepsilon)(K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|)} &+ \frac{K_{|\mathcal{C}|}\sigma_s}{(1+\varepsilon)(K_{|\mathcal{C}|}+\max\{1,\gamma\}|\mathcal{C}|)} < \sigma_s \end{aligned} \tag{28}$$

Since $r_k^c \leq r_k$ for all channels c, $\sum_{b \in \mathcal{C}} r_k^b \geq \sigma_s r_k \geq \sigma_s r_k^c$ for all $c \in \mathcal{C}$. Thus, we obtain that for all links l:

$$\left(\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{\sum_{b \in \mathcal{C}} \overline{x_k^b}}{\sum_{b \in \mathcal{C}} r_l^b} + \sum_{k \in \mathbf{I}'(l)} \frac{\sum_{b \in \mathcal{C}} \overline{x_k^b}}{\sum_{b \in \mathcal{C}} r_k^b} \right) \leq \left(\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \sum_{b \in \mathcal{C}} \frac{\overline{x_k^b}}{\sigma_s r_k} + \sum_{k \in \mathbf{I}'(l)} \sum_{b \in \mathcal{C}} \frac{\overline{x_k^b}}{\sigma_s r_k} \right) = \frac{1}{\sigma_s} \sum_{b \in \mathcal{C}} \left(\sum_{k \in \mathcal{A}(l)} \sum_{d \in \mathcal{C}} \frac{\overline{x_k^d}}{r_k^d} + \sum_{k \in \mathbf{I}'(l)} \frac{\overline{x_k^b}}{r_k^b} \right) < 1$$

using (28)) (29)

When $\lambda_l = 0$ for all l, the queue-lengths are trivially stable. Hence, let us only consider the case where $\lambda_l > 0$ for at least one link $l \in \mathcal{L}$. Let $y_{min} = \min_{l \in \mathcal{L},\ \lambda_l > 0} \frac{\lambda_l}{r_l}$. Let $Q_{init} = \max_{l \in \mathcal{L}} \frac{q_l^c(0)}{r_l^c}$, i.e., Q_{init} is the maximum of the initial normalized queue-lengths. Note that if $\lambda_l = 0$ for some link l, then $\frac{q_l^c(t)}{r_l^c} \leq \frac{q_l^c(0)}{r_l^c} \leq Q_{init}$ for all channels c.

$$
\begin{aligned}
&E[V_q(\overrightarrow{q}(t+1)) - V_q(\overrightarrow{q}(t)) | \overrightarrow{q}(t)] \\
&\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}E[\frac{q_k^d(t+1)-q_k^d(t)}{r_k^d}] + \sum_{k\in\mathbf{I}'(l)}E[\frac{q_k^c(t+1)-q_k^c(t)}{r_k^c}]\right) \\
&+\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}E\left[\frac{(q_l^c(t+1)-q_l^c(t))}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{q_k^d(t+1)-q_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{(q_k^c(t+1)-q_k^c(t))}{r_k^c}\right)\right] \\
&\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}E\left[\frac{\lambda_k^d(t)-x_k^d(t)}{r_k^d}\right] + \sum_{k\in\mathbf{I}'(l)}E\left[\frac{\lambda_k^c(t)-x_k^c(t)}{r_k^c}\right]\right) \\
&\qquad\qquad +2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}E\left[\frac{\lambda_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k^c(t)}{r_k^c}\right)\right] \\
&\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}\left(E\left[\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b}\right] - E\left[\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right]\right) \\
&\qquad\qquad +2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}E\left[\frac{\lambda_l(t)}{\sum_{b\in\mathcal{C}}r_l^b}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b}\right)\right] \\
&\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}\left(E\left[\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k(t)}{\sum_{b\in\mathcal{C}}r_k^b}\right] - E\left[\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right]\right) + C_1 \\
&\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} - E\left[\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right]\right) + C_1 \\
&= 2\sum_{(l,c)\in\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} - E\left[\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right)\right]\right) \\
&\qquad\qquad +2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} - E\left[\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right)\right]\right) + C_1 \\
&\leq 2\sum_{(l,c)\in\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left[\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b}\right) - \left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\sum_{b\in\mathcal{C}}\overline{x_k^c}}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\sum_{b\in\mathcal{C}}\overline{x_k^c}}{\sum_{b\in\mathcal{C}}r_k^b}\right)\right. \\
&+\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\sum_{b\in\mathcal{C}}\overline{x_k^b}}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\sum_{b\in\mathcal{C}}\overline{x_k^b}}{\sum_{b\in\mathcal{C}}r_k^b}\right) - E\left[\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{x_k^d(t)}{r_k^d} + \sum_{k\in\mathbf{I}'(l)}\frac{x_k^c(t)}{r_k^c}\right)\right] \\
&\qquad\qquad \left. +2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b}\right)\right] + C_1 \\
&\leq 2\sum_{(l,c)\in\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left[-\varepsilon\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b}\right)\right] + 2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b} + \sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{\sum_{b\in\mathcal{C}}r_k^b}\right) + C_1 \\
&\qquad\qquad \text{using (25), (21) and (29)}
\end{aligned}
\tag{30}
$$

$$\leq 2 \sum_{(l,c)\in\mathcal{L}'(t)} \frac{q_l^c(t)}{r_l^c}\left[-\varepsilon\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{r_k}+\sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{r_k}\right)\right] - 2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\varepsilon y_{min} + 2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\varepsilon y_{min}$$

$$+2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{r_k}+\sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{r_k}\right)+C_1 \text{ (subtracting and adding back } 2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\varepsilon y_{min})$$

$$\leq 2\sum_{l\in\mathcal{L}}\sum_{c\in\mathcal{C}}\frac{q_l^c(t)}{r_l^c}(-\varepsilon y_{min}) + 2\varepsilon y_{min}\sum_{\substack{l\in\mathcal{L}\\ \lambda_l=0}}\sum_{c\in\mathcal{C}}Q_{init} + 2\varepsilon y_{min}\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c} + 2\sum_{(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)}\frac{q_l^c(t)}{r_l^c}\left(\sum_{k\in\mathcal{A}(l)}\sum_{d\in\mathcal{C}}\frac{\lambda_k}{r_k}+\sum_{k\in\mathbf{I}'(l)}\frac{\lambda_k}{r_k}\right)+C_1$$

(where the second term compensates for including links l having $\lambda_l = 0$ in the first term)

$$\leq -2\varepsilon\frac{y_{min}}{r_{max}}\sum_{l\in\mathcal{L}}q_l(t)+C_3 \tag{31}$$

where $C_1 = \frac{2|\mathcal{L}||\mathcal{C}|\eta(A_{max}|\mathcal{C}|+I_{max})}{(\min_{l\in\mathcal{L}} r_l)^2}$, and $C_3 = C_1 + 2\varepsilon y_{min}|\mathcal{L}||\mathcal{C}|Q_{init} + 2\varepsilon y_{min}|\mathcal{L}||\mathcal{C}| + 2|\mathcal{L}||\mathcal{C}|(A_{max}|\mathcal{C}|+I_{max})$. [3]

Invoking Lemma 2 from [9], this proves stability. ■

[3] Note that $\frac{q_l^c(t)}{r_l^c} < 1$ for all $(l,c)\in(\mathcal{L}\times\mathcal{C})-\mathcal{L}'(t)$, and for any feasible load-vector $\overrightarrow{\lambda}$: $\frac{\lambda_l}{r_l}\leq 1$ for all $l\in\mathcal{L}$