Some Results on Bit/Byte Bounded Codes and Proximity Detecting Codes
(A Brief Note)*

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Abstract

This report presents some new results on two classes of codes: $(t, u)$-bit/byte bounded codes that can handle up to $u$ bytes in error, provided that each byte contains at most $t$ erroneous bits, and $t$-proximity detecting codes that can detect if a received word is within distance $t$ of the transmitted codeword.

1 Introduction

In this report, we consider two types of codes, and present some new results. First class of codes is referred to as the bit/byte-bounded error control codes. $(t, u)$-codes considered here can handle errors in up to $u$ bytes provided that at most $t$ bits in each byte are in error.

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The $(t, u)$-error model is similar to a symmetric error model in [6, 5], but somewhat different from the $t/u$-error model used in our previous work on bit/byte bounded errors [4, 3]. The second class of codes is proximity detecting codes [1].

2 \hspace{0.2cm} \textbf{$(t, u)$-Unidirectional Error Detecting (UED) Codes}

We first derive a lower bound on the number of checkbits needed in a systematic $(t, u)$-UED code.

\textbf{Theorem 1} A systematic $(t, u)$-UED code must use at least $\lceil u \log_2(t + 1) \rceil$ checkbits.

\textbf{Proof:} Consider a data word $D$ that includes $u$ non-zero words such that $t$ bits in each non-zero byte are 1. Also, consider all the data words that are covered by $D$ (a word $W$ is covered by $D$ if $D$ has a 1 in each bit position where $W$ has a 1). There are $(t + 1)^u$ such data words, including $D$ itself. Let the set of these $(t + 1)^u$ data words be called $S$. Since any data word in set $S$ can be changed into any other data word $S$, due to a $t/u$-error, distinct checkbits must be associated with each data word in set $S$. Therefore, at least $\lceil u \log_2(t + 1) \rceil$ are needed.

For $u = 2$, the table below lists the lower bound for various values of $t$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2,3</th>
<th>4</th>
<th>5 $\leq t$ $\leq 7$</th>
<th>8 $\leq t$ $\leq 10$</th>
<th>11 $\leq t$ $\leq 15$</th>
<th>16 $\leq t$ $\leq 21$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower bound</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

\textbf{Optimal $(t, 2)$-UED codes}

References [2, 3] present the design of a $t/1$-unidirectional error correcting code, using a single digit error correcting code, say $C_1$. The basic idea behind this design is to encode the number of non zero bits in each byte, modulo $t + 1$, as the data. If $C_1$ is used as an error correcting code, then it can pinpoint the actual byte (at most one such byte is allowed) in which the number of non-zero bits has changed – this results in the $(t, 1)$-UEL capability\textsuperscript{1}. On the other hand, if $C_1$ is used for error detection, then it can be used to detect up to 2 bytes in which the number of non-zero bits has changed. This yields a $(t, 1)$-UED capability.

\textsuperscript{1}Actually, the codes in [2, 3] are defined to be $t/1$-UEC. In a $t/u$-error, at most $t$ bits in up to $u$ bytes are erroneous. However, note that, $t/u$-errors and $(t, u)$-errors are identical for $u = 1$. 

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The above discussion implies that the \( (t,1) \)-UCE codes in \([2, 3] \) are also \( (t,2) \)-UED. Additionally, it turns out that for several values of \( t \), the \((t,2)\)-UED codes thus obtained match the lower bound on the number of checkbits in the table above. The above design can be easily extended to obtain \((t,u)\)-UED codes by choosing \( C_1 \) to be \( u \)-error detecting code, for the given \( u \).

### 3 Non-Binary Proximity Detecting Codes

Proximity detecting codes \([1] \) are useful to detect when a received word is within a specified distance of the transmitted codeword. In our previous work, we considered design of binary proximity detecting codes. Now, we consider non-binary proximity detecting codes. In this case, each digit in a codeword is non-binary. We assume that the sender and the receiver are connected by a bus, which is initialized to all-0. The sender sends all the digits of the codeword together. At the receiver, all non-zero bits representing a single digit arrive together, however, bits in different bytes take different amounts of time to arrive. (Alternatively, the bus may use multi-valued logic that can carry non-binary digits.)

A \( t \)-digit proximity detecting code \((t\text{-DPD})\) will let the receiver determine whether it has received all but, at most, \( t \) non-zero digits of the codeword.

Let \( X_i \) denote the \( i \)-th digit of \( X \).

**Definition 1** For \( X \) and \( Y \), \( X \subseteq Y \) if and only if, for all \( i \), either \( X_i = 0 \) or \( X_i = Y_i \).

If \( X \subseteq Y \), we say that \( Y \) covers \( X \).

**Definition 2** If (i) \( A \subseteq X \) and \( A \subseteq Y \), and (ii) for any \( B \leq A \), if \( B \subseteq X \) and \( B \subseteq Y \), then \( B \leq A \), then \( A \) is said to be the maximum common subset of \( X \) and \( Y \), and denoted as \( M(X,Y) \).

**Definition 3** Weight of a word \( X \) is the number of non-zero digits in \( X \).

**Theorem 2** A code \( C \) with minimum codeword weight \( t \) is \( t \)-digit proximity detecting \((t\text{-DPD})\) if and only if for any \( X,Y \in C \), such that \( X \neq Y \), one of the following conditions is true: (a) \( N(X,M(X,Y)) = N(Y,M(X,Y)) \leq t \) or (b) \( N(X,M(X,Y)) > t \) and \( N(X,M(X,Y)) > t \).
Proof: This theorem generalizes a result previously obtained for binary codes [1]. The proof for the theorem is obtained by generalizing a proof in [1]. □

Corollary 1 Constant weight codes are t-DPD for all values of t.

Proof: In a constant weight code, weight of each codeword is identical, say W. Since weight(X) = weight(Y) = W, if X and Y belong to the constant weight code, it follows that N(X, M(X, Y)) = N(Y, M(X, Y)). Therefore, by Theorem 2, the code is t-DPD for any t ≤ W. Also, if t > W, then t exceeds weight of every codeword in the constant weight code – in this case, the code is trivially t-DPD. □

4 Summary

This report presents some new results on bit/byte codes and proximity detecting codes.

References


