A Distributed Throughput-Optimal CSMA/CA

Tae Hyun Kim, Jian Ni and Nitin H. Vaidya
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Coordinated Science Laboratory
Dept. Electrical and Computer Engineering in the University of Illinois at Urbana-Champaign
Email: {tkim56, jianni, nhv}@illinois.edu

***Abstract*—This paper addresses a distributed throughput-optimal CSMA/CA for wireless ad hoc networks, which is called the preemptive CSMA/CA. It achieves the optimality in a completely distributed fashion, even with discrete backoff time, non-zero carrier sense delay and data packet collisions. The algorithm is derived from the extension of Q-CSMA by Ni, Bo and Srikant to include collisions and is similar to the Jiang and Walrand's CSMA/CA. The analysis on the preemptive CSMA/CA provides with the understanding on the relationship among the throughput-optimal CSMA/CAs in the literature.**

I. Introduction

Since Tassiulas and Ephremides first introduced the notion of throughput-optimality in [1] for wireless multi-hop networks, there has been a large body of research on the scheduling algorithms that achieve the optimality. Due to the complexity of the max-weight algorithm proposed in [1], several low-complexity alternatives such as maximal scheduling and greedy maximal scheduling have been studied, but in general these algorithms can only achieve a fraction of the capacity region (e.g., [2]).

Recently with a surprise, it has been shown that a simple carrier sense multiple access with collision avoidance (CSMA/CA) algorithm can achieve the throughput-optimality in a completely distributed fashion [3]–[8]. The main leverage is to utilize the Glauber dynamics to solve a maximum weight independent set problem in a distributed manner.

However, the algorithms achieve the optimality with ideal assumptions such as continuous-time backoff time, zero carrier sense delay and no collision. These assumptions basically eliminate the loss of the CSMA/CA algorithms, which is not the case in practice.

This paper addresses a distributed, throughput-optimal CSMA/CA for wireless ad hoc networks, which is named as the preemptive CSMA/CA. It is completely distributed in a sense that it only requires carrier sense results from the outside of a link (or a node) and it runs with low complexity. At the same time, our CSMA/CA is practical enough to achieve the optimality even with discrete backoff time, non-zero carrier sense delay and data packet collisions.

In essence, our preemptive CSMA/CA resembles the CSMA/CAs in [8] and [4]. We basically extends Q-CSMA in [4] to include data packet collisions. Our CSMA/CA works similarly to the CSMA/CA in [8], but the analytic framework is mostly based on that used in [4]. As a result, the analysis is simpler and the algorithm covers a wider class of weight functions embedded in the CSMA/CA than that in [8]. Moreover, it directly uses the queue length information, which is also inherited from Q-CSMA.

Our preemptive CSMA/CA in fact bridges the gap between two throughput-optimal CSMA/CAs, which has not been well understood. Furthermore, this work identifies the key strategy for the throughput-optimality, which can also be observed in the ideal CSMA/CAs in [3], [5]–[7].

II. Model

We model a single channel wireless ad hoc network by a graph $G = (V, E)$ where V is the set of nodes and E is the set of links. For ease of exposition, we describe the model with links (link-centric model), which can be easily transferred to a node-centric model. This graph based model is also used in [3]–[8].

The access to the wireless medium is time slotted, which is indexed by nonnegative integer t. For a given time t, we denote a link rate by a vector $\mathbf{x}(t)$ of which elements are $x_i(t)$, $i \in E$. Without loss of generality, these link rates are all normalized and are either 0 or 1 for simplicity. Thus, a link rate $\mathbf{x}(t)$ also represents a schedule. With a little abuse of notation, we also use $\mathbf{x}(t)$ as a set and write $i \in \mathbf{x}(t)$ if $x_i(t) = 1$. We consider one-hop traffic only, but one may want to incorporate a congestion control algorithm to ours for multi-hop networking as done in [3].

We model the interference in the wireless networks by conflict relationships among the links. Let us denote the set of conflict links of a link i as C_i, $i \in E$. When the link i transmits, if one or more links in the set C_i are active at the same time, the transmission will fail. Furthermore, it is assumed that the conflict relationship is symmetric; if $i \in C_j$, then $j \in C_i$. A feasible schedule $\mathbf{x}$ is defined as a schedule that has no active link in the set of conflict links ($\sum_{j \in C_i} x_j = 0$ for $i \in \mathbf{x}$). All feasible schedules comprise the feasible schedule set, which is denoted by $\mathcal{F}$.

Carrier sense is modeled to be performed at the end of each slot in our model as depicted in Figure 1.[1] The sensing lasts

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[1]The model is just for analytical convenience since it is equivalent to have carrier sense performed at the beginning of each slot.

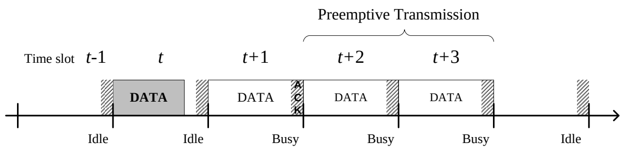


Fig. 1. Carrier sense model. Carrier sense observes the later part of a time slot. Under time line, it is shown the results of the carrier sense given to one of conflict neighbors of the transmitting link.

for α duration and its result is used to decide if the immediate next slot is allowed to be accessed. The current transmitting link thus may fill up the slot if it wants to use the next one and leaves the last α duration idle if not. For instance, in Figure 1, time slot $t+1$ is completely filled with an acknowledgement (ACK) packet from the receiver and slots $t+2$ and $t+3$ are filled by longer data packets, all for the preemptive medium access. We define as a *preemptive transmission* the transmission that preempts the medium access of others by making the α duration busy. By definition, the first packet in success at time $t+1$ in Figure 1 is not included in the preemptive transmission. *It is important to notice that our carrier sense model can detect the preemptive transmission only,* which is also true for CSMA/CAs in [4], [8].

We assume that the carrier sense works ideally, which means that a link i detects any transmission by any link in C_i whenever there is. Such ideal carrier sense eliminates the possibility of collisions once the link preempts the access of the conflict neighbors.

The *capacity region* of a network is the set of all arrival rates $\boldsymbol{\lambda}$ for which there exists a scheduling algorithm that can stabilize the queues in the network. The capacity region can be characterized as follows (e.g., [1]):

$$\Lambda = \{\boldsymbol{\lambda} \mid \exists \boldsymbol{\mu} \in Co(\mathcal{F}) : \mathbf{0} \leq \boldsymbol{\lambda} < \boldsymbol{\mu}\}, \tag{1}$$

where $Co(\mathcal{F})$ is the convex hull of the set of feasible schedules in $\mathcal{F}$. When dealing with vectors, inequalities are interpreted component-wise. A scheduling algorithm is said to be *throughput-optimal* if it can keep the network queues stable for all arrival rates in the capacity region Λ.

Notations: We use $\|\cdot\|$ for usual Euclidean norm operation. An over-bar on a variable indicates complementary probability, for example, $\bar{a}_i = 1 - a_i$. We shorten the notation such as $P(a|b) := P(a(t) = a|b(t) = b)$ and $P(a|a') := P(a(t) = a|a(t-1) = a')$.

III. Preemptive CSMA/CA

We introduce the preemptive CSMA/CA in this section. It is called the *preemptive* CSMA/CA as the defining feature is that some links preempt the medium access of others. The preemption for the medium access is to the extent of one link's conflict link set, C_i by our carrier sense model.

The preemptive CSMA/CA can be implemented in a completely distributed way. The only information that needs to come from the outside of a link is the result of the carrier sense operation.

Algorithm 1 Preemptive CSMA/CA for link i at time t

1: /* Ber(p): Bernoulli trial with prob. p */
2:
3: If $\sum_{j \in C_i} u_j(t-1) = 0$ $\triangleright$ *Ideal carrier sense*
4: If $u_i(t-1) = 1$
5: $x_i(t) \leftarrow$ Ber(p_i)
6: Else
7: $x_i(t) \leftarrow$ Ber(a_i)
8: Else
9: $x_i(t) = 0$
10:
11: Transmit packets by schedule $\mathbf{x}(t)$
12: Update

$$u_i(t) = \begin{cases} 1 & \text{if } x_i = 1 \text{ and successful,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

A. Operations

The details of the proposed CSMA/CA are shown in Algorithm 1. The function Ber(p) returns 1 with probability p and 0 with $1-p$.

By the preemptive CSMA/CA in Algorithm 1 a link maintains two internal state variables: transmission schedule $x_i(t)$ and preemption $u_i(t)$. At the beginning of a time slot, a link refers to the report from the carrier sense at the previous slot. If there was no preemptive transmission from other conflict neighbors (Line 3), the link observes $u_i(t-1)$ to see if it has been preempting the medium access of its conflict neighbors. If so, it selects with probability p_i that it continue to preempt the access of other links by another transmission (Line 4–5). Thus, the preemptive transmission will be finished only when the link i decides so, and no interference can be generated to the preemptive transmission under the ideal carrier sense assumption.

When the link that has preempted others decides not to preempt them any more by drawing 0 on Line 5, the slot t is left idle since all neighbors will not access the medium due to its carrier sense busy status at time $t-1$. Essentially, the idle slot is used to signal other neighbor links in conflict that the medium is released from the preemption.[2]

By Line 7, the links that have not preempted the neighbors ($u_i(t-1) = 0$) compete only when the medium in the previous slot was free from the preemptive transmission (Line 3). If free, the links set $x_i(t) = 1$ with probability a_i.

After $x_i(t)$'s are updated, the links in the networks transmit a packet and/or observe the medium according to $x_i(t)$. If $x_i(t) = 1$, a packet is transmitted, and the transmission result is monitored to update $u_i(t)$ by (2). If the transmission is successful, which is typically informed by an ACK packet reception, it obtains the preemption for the medium access by having $u_i(t) = 1$. Otherwise, it observes the medium to see if the medium access to the next slot is preempted by one of others.

[2]The idle slot would have been used by the link that has preempted the others, but we do not allow it in this paper for analytical tractability.

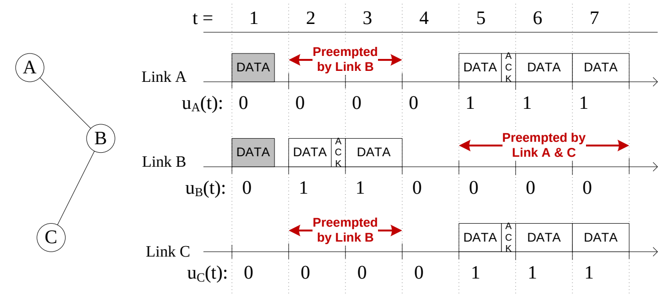


Fig. 2. An example of the progress of internal variables by the preemptive CSMA/CA. The left is the conflict graph of the considered network. The gray boxes indicate the packets in collision.

Figure 2 shows one example of the schedules by the preemptive CSMA/CA. The network has three links, A, B and C where A and B conflict and so do B and C. In the first slot link A and B transmit at the same time, resulting in a collision. In the second slot, link B succeeds in a transmission and thus, preempts the medium access of links A and C at the third slot. In the fourth slot, link B decides not to preempt others any more, leaving that slot idle. As discussed, that idle slot cannot be used by other links, either. In the fifth slot, link A and C transmit and succeed together as they do not conflict with each other.

B. Dynamics

We model the dynamics of Algorithm 1 by a discrete-time Markov chain (DTMC). For ease of derivation, instead of $\mathbf{x}(t)$, the set of links that are in collision is now part of the state variables, which is denoted by $\mathbf{y}(t)$.

In our interference model, the collision happens when two transmissions are within each other's conflict link set. For $\mathbf{y}$ to be valid, for link $i \in \mathbf{y}$, there should be at least one link in transmission in C_i. This condition is specified with an aid of the following function:

$$\phi_{\mathbf{y}}(i) := \begin{cases} 0 & \text{if } \sum_{j\in C_i} y_j \neq 0, \\ 1 & \text{otherwise,} \end{cases} \qquad (3)$$

by which the condition for $\mathbf{y}$ is that for $i \in \mathbf{y}$, $\phi_{\mathbf{y}}(i) = 0$.

For a given $\mathbf{u}(t)$, there could be multiple $\mathbf{y}(t)$ that lead to the same $\mathbf{u}(t)$ by definition. Denote by $\mathcal{Y}_{\mathbf{u}|\mathbf{u}'}$ the set of $\mathbf{y}(t)$'s that result in the same $\mathbf{u}$ for the given $\mathbf{u}'$. The definition of the set is

$$\mathcal{Y}_{\mathbf{u}|\mathbf{u}'} := \left\{\mathbf{y} \subseteq E \setminus (\mathbf{u}\cup\mathbf{u}') \setminus C_{\mathbf{u}\cup\mathbf{u}'} : \phi_{\mathbf{y}}(i) = 0 \text{ for } i \in \mathbf{y}\right\},$$

where $C_{\mathbf{u}\cup\mathbf{u}'} := \cup_{j\in(\mathbf{u}\cup\mathbf{u}')} C_j$. The definition is by the fact that the preemptive transmission is not collided (interfered) by the transmissions of conflict neighbors in the ideal carrier sense case. The set includes the empty set by definition, and it is the only element if either $\mathbf{u}$ or $\mathbf{u}'$ is maximal. Note that the order of $\mathbf{u}'$ and $\mathbf{u}$ is not relevant. We use notation $\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}$ for $\mathcal{Y}_{\mathbf{u}|\mathbf{u}'}$ to explicitly show this property.

With the set $\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}$, the transition probability can be written with respect to $\mathbf{u}(t)$ as

$$P(\mathbf{u}|\mathbf{u}') = \sum_{\mathbf{y}\in\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}} P(\mathbf{u},\mathbf{y}|\mathbf{u}'), \qquad (4)$$

which gives the DTMC with state $\mathbf{u}(t)$ only. The dynamics of this Markov chain is described by the following transition probability.

Lemma 1. *The transition probability from $\mathbf{u}'$ to $\mathbf{u}$ is*

$$P(\mathbf{u}|\mathbf{u}') = \prod_{i\in\mathbf{u}\cap\mathbf{u}'} p_i \prod_{j\in\mathbf{u}'\setminus\mathbf{u}} \bar{p}_j \prod_{l\in\mathbf{u}\setminus\mathbf{u}'} a_l \prod_{\substack{k\in E\setminus C_{\mathbf{u}'}\\ \setminus(\mathbf{u}\cup\mathbf{u}')}} \bar{a}_k \sum_{\mathbf{y}\in\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}} \prod_{y\in\mathbf{y}} \frac{a_y}{\bar{a}_y} \qquad (5)$$

if $\mathbf{u}\cup\mathbf{u}' \in \mathcal{F}$, and $P(\mathbf{u}|\mathbf{u}') = 0$ otherwise.

Proof: By (2), $\mathbf{u}$ and $\mathbf{u}'$ are all in $\mathcal{F}$. Suppose $\mathbf{u}\cup\mathbf{u}' \notin \mathcal{F}$. Then, there exists at least one link i which satisfies $u'_i = 1$ and $\sum_{j\in C_i} u'_j = 0$ at time $t-1$ and $u_i = 0$ and $\sum_{j\in C_i} u_j = 1$ at time t. However, this cannot happen since there always exists one idle slot when a preemptive transmission finishes ($u'_i = 1 \rightarrow u_i = 0$). Thus, to have a non-zero transition probability, $\mathbf{u}\cup\mathbf{u}' \in \mathcal{F}$ should hold.

Now suppose $\mathbf{u}\cup\mathbf{u}' \in \mathcal{F}$. The transition probability can be calculated from $P(\mathbf{u},\mathbf{y}|\mathbf{u}')$ by (4). $P(\mathbf{u},\mathbf{y}|\mathbf{u}')$ is

$$P(\mathbf{u},\mathbf{y}|\mathbf{u}') = \prod_{i\in\mathbf{u}\cap\mathbf{u}'} p_i \prod_{j\in\mathbf{u}'\setminus\mathbf{u}} \bar{p}_j \prod_{l\in(\mathbf{u}\setminus\mathbf{u}')\cup\mathbf{y}} a_l \prod_{\substack{k\in E\setminus C_{\mathbf{u}'}\\ \setminus\mathbf{y}\setminus(\mathbf{u}\cup\mathbf{u}')}} \bar{a}_k, \qquad (6)$$

which is derived by the following case-by-case analysis.

- Consider the links that have sent packets with the preemption in time slot $t-1$. Here are two possible transitions.
  - Link $i \in \mathbf{u}\cap\mathbf{u}'$: it transmits a packet with probability p_i at time t, which is due to Line 5.
  - Link $j \in \mathbf{u}'\setminus\mathbf{u}$: it does not transmit at time t with $\bar{p}_j$ probability, giving up the preemption.
- Links that have sensed no preemption transmission in time slot $t-1$ may transmit packets at time t with a_l probability. There are two possible subsets of such links.
  - Link $l \in \mathbf{y}$: it experiences collision at time t.
  - Link $l \in \mathbf{u}\setminus\mathbf{u}'$: it successfully transmits its packet at time t, obtaining the preemption.
- Consider the links that we have not examined, which are all silent at time t.
  - Link $m \in C_{\mathbf{u}'}$: it should be silent at time t since it sensed the preemptive transmission at the end of time slot $t-1$, which corresponds to $\prod_{m\in C_{\mathbf{u}'}} 1$. For simplicity, we do not include this in (6).
  - Link $k \in E\setminus C_{\mathbf{u}'}\setminus\mathbf{y}\setminus(\mathbf{u}\cup\mathbf{u}')$: it is silent in time slot t with $\bar{a}_k$ probability.

By multiplying the probabilities for transitions, we have (6). Putting (6) into (4), we have (5). This completes the proof. ■

One may find (5) complicated, but it is simply a product of access probabilities from different sets of links. Two access probabilities, p_i and a_i, are used in the preemptive CSMA/CA, and thus, the links in a network are classified into four groups that are associated with p_i, $\bar{p}_i$, a_i and $\bar{a}_i$, respectively.

The stationary distribution of the chain is easily obtained by the reversibility of the chain as follows.

Lemma 2. *The DTMC $\mathbf{u}(t)$ is reversible, which has the following unique stationary distribution: if $\mathbf{u} \in \mathcal{F}$*

$$\pi(\mathbf{u}) = \frac{1}{z}\left(\prod_{i\in\mathbf{u}} \frac{a_i}{\bar{p}_i}\right)\left(\prod_{k\in C_{\mathbf{u}}} \bar{a}_k\right), \tag{7}$$

where z is the normalization constant, and $\pi(\mathbf{u}) = 0$ otherwise.

Proof: We are going to show that the transition probability (5) and the given stationary distribution (7) satisfy the detailed balance equation. Suppose $\mathbf{u} \cup \mathbf{u}' \in \mathcal{F}$. Otherwise, $P(\mathbf{u}|\mathbf{u}') = 0$ and the detailed balance equation holds trivially.

The set $(\mathbf{u} \cup \mathbf{u}')$ and $\mathbf{C}_{\mathbf{u}'}$ are disjoint by the condition $\mathbf{u} \cup \mathbf{u}' \in \mathcal{F}$. Also, from the definition of $\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}$, $(\mathbf{u} \cup \mathbf{u}')$ and $\mathbf{y}$ are disjoint, and so do $C_{\mathbf{u}'}$ and $\mathbf{y}$. These facts allow us to rewrite the transition probability (5) as

$$\begin{aligned} P(\mathbf{u}|\mathbf{u}') = \prod_{i\in\mathbf{u}\cap\mathbf{u}'} p_i \left(\frac{\prod_{j\in\mathbf{u}'\cup\mathbf{u}} \bar{p}_j}{\prod_{j\in\mathbf{u}} \bar{p}_j}\right)\left(\frac{\prod_{l\in\mathbf{u}'\cup\mathbf{u}} a_l}{\prod_{l\in\mathbf{u}'} a_l}\right) \\ \cdot \left(\frac{\prod_{k\in E\setminus(\mathbf{u}\cup\mathbf{u}')} \bar{a}_k}{\prod_{k\in C_{\mathbf{u}'}} \bar{a}_k}\right) \sum_{\mathbf{y}\in\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}} \prod_{y\in\mathbf{y}} \frac{a_y}{\bar{a}_y}. \end{aligned} \tag{8}$$

The detailed balance equation, $\pi(\mathbf{u}')P(\mathbf{u}|\mathbf{u}') = \pi(\mathbf{u})P(\mathbf{u}'|\mathbf{u})$, is checked by considering

$$\frac{P(\mathbf{u}|\mathbf{u}')}{P(\mathbf{u}'|\mathbf{u})} = \frac{\prod_{j\in\mathbf{u}'} \bar{p}_j \prod_{l\in\mathbf{u}} a_l \prod_{k\in C_{\mathbf{u}}} \bar{a}_k}{\prod_{j\in\mathbf{u}} \bar{p}_j \prod_{l\in\mathbf{u}'} a_l \prod_{k\in C'_{\mathbf{u}}} \bar{a}_k} \tag{9}$$

$$= \frac{\frac{1}{z}\prod_{j\in\mathbf{u}} \frac{1}{\bar{p}_j} \prod_{l\in\mathbf{u}} a_l \prod_{k\in C_{\mathbf{u}}} \bar{a}_k}{\frac{1}{z}\prod_{j\in\mathbf{u}'} \frac{1}{\bar{p}_j} \prod_{l\in\mathbf{u}'} a_l \prod_{k\in C'_{\mathbf{u}}} \bar{a}_k} = \frac{\pi(\mathbf{u})}{\pi(\mathbf{u}')}, \tag{10}$$

where (9) is obtained by canceling out the product of p_i's and the summations over $\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'}$. The cancelation of the summations is due to $\mathcal{Y}_{\mathbf{u}\cup\mathbf{u}'} = \mathcal{Y}_{\mathbf{u}'\cup\mathbf{u}}$, which is true by definition. Thus, the detailed balance equation is satisfied by the stationary distribution given by (7). Therefore, the chain is indeed reversible, and (7) is the unique stationary distribution. ∎

The stationary distribution (7) is compact and illustrates the steady state behavior very well; a successful transmission happens only when conflict neighbors are all silent, and such transmission is initiated by the medium access with probability a_i. Once the transmission is successful, obtaining the preemption, the preemptive transmission continues for $1/\bar{p}_i$ time slots in a mean sense without interruption. Notice that $1/\bar{p}_i$ is the mean of the geometric distribution with success probability $\bar{p}_i$.

C. Performance

For the set of fixed a_i and p_i over time, the preemptive CSMA/CA would reach the steady state described by (7). If the probabilities a_i and p_i dynamically change in the transient state, the performance analysis becomes challenging due to the memory across time slots. Instead of the transient state analysis, the following assumption is made.

Assumption 1 (Time-Scale Separation)**.** *The chain $\mathbf{u}(t)$ immediately converges to the steady state in every time slot.*

This is essentially to separate the time-scale of the dynamics of scheduling and queue lengths; by the assumption, the queue length varies slow enough for the Markov chain to see no change of it until the chain converges to the steady state. Recently, the assumption has been relaxed for the ideal CSMA/CA cases in [6], [7].

Under this assumption, the preemptive CSMA/CA can be proven to be throughput-optimal with the properly chosen p_i and a_i. The proof relies on the result in [9]. Denote q_i as the queue length of a link i and vector $\mathbf{q}$ as the lengths of all links. Consider to use weights that are function of the queue lengths, $w_i(q_i(t))$ where $w_i : [0,\infty] \to [0,\infty]$ are functions that satisfy the following conditions:

1) $w_i(q_i)$ is a nondecreasing, continuous function with $\lim_{q_i\to\infty} w_i\big(q_i(t)\big) = \infty$.
2) Given any $\alpha_1 > 0$, $\alpha_2 > 0$ and $0 < \epsilon < 1$, there exists $\beta < \infty$, such that for all $q_i > \beta$ and $i \in E$, we have

$$\begin{aligned} (1-\epsilon)w_i(q_i) \le& w_i(q_i - \alpha_1) \\ &\le w_i(q_i + \alpha_2) \le (1+\epsilon)w_i(q_i). \end{aligned} \tag{11}$$

Theorem 1 (Eryilmaz, Srikant and Perkins [9])**.** *For a scheduling policy, if given any ϵ and δ, $0 < \epsilon, \delta < 1$, there exists a $\beta > 0$ such that: in any time slot t, with probability greater than $1-\delta$, the scheduling algorithm chooses a schedule $\mathbf{x}(t) \in \mathcal{F}$ that satisfies*

$$\sum_{i\in\mathbf{x}(t)} w_i(t) \ge (1-\epsilon)\max_{\mathbf{x}\in\mathcal{F}} \sum_{i\in\mathbf{x}} w_i(t) \tag{12}$$

whenever $\|\mathbf{q}(t)\| > \beta$. Then the scheduling policy is throughput-optimal.

Using this, the throughput-optimality of the preemptive CSMA/CA is proven. First notice that $\mathbf{u}(t)$ is the links in successful transmissions at time t. Before the proof, we denote a maximum weight schedule by $\mathbf{u}^*(t) := \arg\max_{\mathbf{u}\in\mathcal{F}} \sum_{i\in\mathbf{u}} w_i(t)$ for a given time t.

Proposition 1. *Choose $\bar{p}_i = \frac{1}{e^{w_i(t)}}$ where $w_i(t)$ is an appropriate function of the queue length as discussed in the above.*[3] *Given ϵ, $0 < \epsilon < 1$ and time t, define*

$$\mathcal{U}(t) := \left\{\mathbf{u} \in \mathcal{F} : \sum_{i\in\mathbf{u}} w_i(t) < (1-\epsilon)\sum_{i\in\mathbf{u}^*(t)} w_i(t)\right\}, \tag{13}$$

and

$$\xi(t) := \sum_{\mathbf{u}\in\mathcal{U}(t)} \frac{\prod_{i\in\mathbf{u}} a_i \prod_{j\in C_{\mathbf{u}}} \bar{a}_j}{\prod_{i\in\mathbf{u}^*(t)} a_i \prod_{j\in C_{\mathbf{u}^*(t)}} \bar{a}_j} \tag{14}$$

[3]Choosing $p_i = \frac{e^{w_i(t)}}{1+e^{w_i(t)}}$ gives the same result with a similar proof.

for time t. Choose a_i such that $\xi(t) < \xi_{\max}$ for all t for some $0 < \xi_{\max} < \infty$. For such p_i and a_i, the preemptive CSMA/CA with the ideal carrier sense is throughput-optimal.

Proof: Define $\pi(\mathcal{U}) := \sum_{\mathbf{u}\in\mathcal{U}} \pi(\mathbf{u})$. We will show that there exists β such that $\pi(\mathcal{U}) < \delta$ whenever $\|\mathbf{q}\| > \beta$. For simpler notation, we use $w_{\mathbf{u}} := \sum_{i\in\mathbf{u}} w_i$.

If we choose $\bar{p}_i = \frac{1}{e^{w_i(t)}}$, the stationary distribution is

$$\pi(\mathbf{u}) = \frac{1}{z}\prod_{i\in\mathbf{u}}\frac{1}{\bar{p}_i}\cdot a_i \prod_{j\in C_{\mathbf{u}}}\bar{a}_j = \frac{1}{z}\prod_{i\in\mathbf{u}} e^{w_i}\cdot a_i \prod_{j\in C_{\mathbf{u}}}\bar{a}_j, \quad (15)$$

and

$$\pi(\mathcal{U}) = \frac{1}{z}\sum_{\mathbf{u}\in\mathcal{U}}\left(e^{w_{\mathbf{u}}}\prod_{i\in\mathbf{u}} a_i \prod_{j\in C_{\mathbf{u}}}\bar{a}_j\right) < \frac{1}{z}e^{(1-\epsilon)w_{\mathbf{u}^*}}\sum_{\mathbf{u}\in\mathcal{U}}\left(\prod_{i\in\mathbf{u}} a_i \prod_{j\in C_{\mathbf{u}}}\bar{a}_j\right) \quad (16)$$

where $e^{w_{\mathbf{u}}}$ for $\mathbf{u}\in\mathcal{U}(t)$ is upper bounded by $e^{(1-e)w_{\mathbf{u}^*}}$ by the definition of $\mathcal{U}(t)$.

A lower bound for z is

$$z = \sum_{\mathbf{u}\in\mathcal{F}}\left(e^{w_{\mathbf{u}}}\prod_{i\in\mathbf{u}} a_i \prod_{j\in C_{\mathbf{u}}}\bar{a}_j\right) > e^{w_{\mathbf{u}^*}}\prod_{i\in\mathbf{u}^*} a_i \prod_{j\in C_{\mathbf{u}^*}}\bar{a}_j, \quad (17)$$

which gives

$$\pi(\mathcal{U}) < \frac{e^{(1-\epsilon)w_{\mathbf{u}^*}}}{e^{w_{\mathbf{u}^*}}}\sum_{\mathbf{u}\in\mathcal{U}}\frac{\prod_{i\in\mathbf{u}} a_i\prod_{j\in C_{\mathbf{u}}}\bar{a}_j}{\prod_{i\in\mathbf{u}^*} a_i\prod_{j\in C_{\mathbf{u}^*}}\bar{a}_j} = e^{-\epsilon w_{\mathbf{u}^*}}\xi(t) < e^{-\epsilon w_{\mathbf{u}^*}}\xi_{\max} \quad (18)$$

Thus, to have $\pi(\mathcal{U}) < \delta$,

$$w_{\mathbf{u}^*} > \frac{1}{\epsilon}\left(\log\frac{1}{\delta} + \log\xi_{\max}\right) \quad (19)$$

should satisfy for any given δ and ϵ, when $\|\mathbf{q}\| > \beta$ for some $\beta > 0$. Therefore, there exists a large $w_{\mathbf{u}^*}$ that satisfies (19) as w_i is a continuous, non-decreasing function of the queue length with $\lim_{\|\mathbf{q}\|\to\infty} w_{\mathbf{u}^*} = \infty$, and thus, $\pi(\mathcal{U}) < \delta$. By Theorem 1, the preemptive CSMA/CA with the ideal carrier sense is throughput-optimal if $\bar{p}_i = 1/e^{w_i(t)}$ and a_i's are chosen such that $\xi(t) < \xi_{\max}$ for all t. ■

The key strategy for the throughput-optimality of the preemptive CSMA/CA is to lengthen the duration of the preemptive transmission proportional to the queue lengths. As the network gets more congested, such duration becomes longer and longer, and the loss by CSMA/CA such as collisions and backoff is relatively small. Thus, in the extremely congested state, the loss is virtually zero and the optimality is achieved.

Our preemptive CSMA/CA explicitly takes into account the loss by random backoff and data packet collisions. The loss by non-zero time for the carrier sense, which corresponds to α out of each preemptive transmission, is not considered for establishing the optimality. However, as the preemptive transmission gets longer, the fraction α to the length of the preemptive transmission becomes negligible. Moreover, the time may be used to receive an ACK packet from the receiver as shown in Figure 1 and 2 in practice.

The range of possible $w_i(t)$ is quite wide for the throughput-optimality. One may want to $w_i = \log\log q_i$ so that the time-scale separation assumption is virtually satisfied, which is the choice made in [6], [7] for their CSMA/CAs to prove the optimality without the assumption. Such choice, however, may yield a poor delay performance.

The choice of the access probability a_i, which is used for non-preemptive transmission, also determines the delay performance while any choice such that $\xi(t) < \xi_{\max}$ for all t achieves the throughput-optimality. We conjecture that there is a_i that minimizes the delay because too small a_i incurs a large delay and too large a_i makes frequent collision happen. The characterization of the delay performance with respect to a_i is out of scope of this paper.

IV. Discussion

The key strategy for the throughput-optimality in our CSMA/CA is the preemptive transmission. This section compares our CSMA/CA with other throughput-optimal CSMA/CAs and explains that the same strategy is shared by all throughput-optimal CSMA/CAs.

A. Relationship to Other CSMA/CAs

1) Q-CSMA [4]: Q-CSMA proposed in [4] has control and data phases in a synchronized manner. Q-CSMA updates the schedule for each slot at random as the preemptive CSMA/CA does, but *only a feasible set of links consider to update their schedules* whereas the preemptive CSMA/CA lets all links consider to update theirs. This feasibility condition is ensured by the control packet exchanges in the control phase. Due to this fact, Q-CSMA requires to have the control phase, which is part of the loss of achievable throughput. On the other hand, the condition enables Q-CSMA to avoid any data packet collisions in the data phase.

Our preemptive CSMA/CA eliminates the need for the control phase, which basically relaxes the feasibility condition for the links to update their schedules in Q-CSMA. In the preemptive CSMA/CA, all links in the network consider to update their schedules according to the carrier sense results, which results in data packet collisions with non-zero probability. Despite the collisions, the throughput-optimality is still achievable. However, to prevent the loss from being significant, we have to introduce another access probability a_i in addition to p_i.

2) CSMA/CA in [8]: Jiang and Walrand proposed a CSMA/CA in [8] of which behavior more resembles that of IEEE 802.11. With their CSMA/CA, the links will access the medium with probability a_i whenever there is no transmission from their conflict link sets. The scheme to achieve the throughput-optimality in Jiang's CSMA/CA is to lengthen the data packet duration exponentially proportional to the estimate of the queue length dynamics.

Regarding the loss by randomness, Jiang's CSMA/CA experiences collisions due to random access, which is the key difference from Q-CSMA. The loss by the collisions is, however, minimized by employing a small control packet that precedes a data packet. Also, it is implicitly assumed that carrier sense reports a positive at the beginning of the slot without a sensing delay if there is an ongoing *successful* transmission in the current slot. Our preemptive CSMA/CA is without such assumption, having one idle slot that follows each preemptive transmission. The idle slots are due to the scheduling decisions made in a per slot basis.

Except for the subtle difference, our preemptive CSMA/CA works similarly to Jiang's. Despite the similarities, however, ours has its own unique values over Jiang's CSMA/CA. The proof for the optimality is simpler and covers a fairly large class of distributions for the packet lengths as well as the exponentially distributed packet lengths (see the conditions for $w_i(t)$ in Section III-C). Also, the preemptive CSMA/CA directly uses queue length information rather than estimating arrival and service rates as Jiang's CSMA/CA. Furthermore, our CSMA/CA relates Jiang's CSMA/CA to Q-CSMA of which relationship has not been well understood.

B. Strategy in Common for Optimality

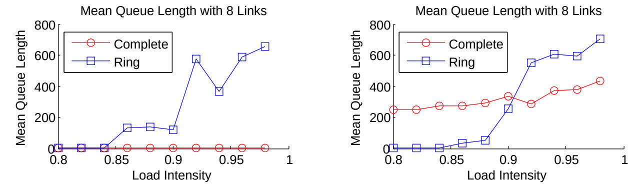
As discussed, the preemptive CSMA/CA achieves the throughput-optimality by successful transmissions of which duration is proportional to the queue length. The carrier sense prevents the other conflict neighbors from interfering with the preemptive transmission. As the network gets more congested, such transmission becomes extremely long and thus, the loss from random access such as collision and backoff becomes negligible. In sum, the key for the optimality is the *preemptive transmission*.

This preemptive transmission is also observed in other CSMA/CAs. In Q-CSMA, a transmission schedule is kept to be the same until the current transmitting link decides to finalize its back-to-back transmission. Jiang's CSMA/CA predetermines the transmission length exponentially proportional to the estimate of the queue length dynamics, and so does the ideal continuous-time CSMA/CA [3]–[6]. Therefore, the same strategy to achieve the throughput-optimality is shared by all throughput-optimal CSMA/CAs.

V. Numerical Results

In Figure 3, we show the expected queue lengths by the preemptive CSMA/CA in two different network topologies with 8 links: one is a fully connected network (complete conflict graph) and the other is a ring network (ring conflict graph). The load intensity is defined as the ratio of arrival rate to the capacity of each network. The queue lengths of all links are averaged out in the figure since they all have the same dynamics. The weight function $w_i = q_i$ is used for the simulations.

Two observations are made from Figure 3. First, the preemptive CSMA/CA in any case stabilizes the networks with the load intensity close to one. However, the expected queue



(a) Access probability $a_i = 0.2$ (b) Access probability $a_i = 0.8$

Fig. 3. Queue lengths with varying load intensity and access probability a_i.

lengths become larger as the intensity gets closer to one. Second, the access probability a_i which has a wide range of choice for the throughput-optimality gives a radically different delay performance according to the conflict graph of the network. For both $a_i = 0.2$ and 0.8, the ring network experiences a similar amount of delay. In contrast, the fully connected network shows a big difference in delay according to a_i.

VI. Conclusion

We have proposed the preemptive CSMA/CA, which is completely distributed and throughput-optimal. The algorithm is obtained by extending Q-CSMA which has no data collision. Although the preemptive CSMA/CA resembles the CSMA/CA from [8], we leverage a different analytical framework to show the throughput-optimality, which is mainly due to [4]. This approach leads us to well understand the relationship of two CSMA/CAs and further most of all throughput-optimal CSMA/CAs in the literature; the key for the optimality is to have the preemptive transmission of which duration is proportional to the queue lengths, and it gives the throughput-optimality even with the loss by the random nature of CSMA/CA.

References

[1] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Automatic Control*, vol. 37, no. 12, pp. 1396–1948, Dec. 1992.

[2] M. Leconte, J. Ni, and R. Srikant, "Improved bounds on the throughput efficiency of greedy maximal scheduling in wireless networks," in *Proceedings of ACM MOBIHOC*, May 2009.

[3] L. Jiang and J. Walrand, "A distributed CSMA algorithm for throughput and utility maximization in wireless networks," in *Proceedings 46th Annual Allerton Conference on Communication, Control and Computing*, September 2008.

[4] J. Ni and R. Srikant, "Distributed CSMA/CA algorithms for achieving maximum throughput in wireless networks," in *Information Theory and Applications Workshop*, Feburary 2009.

[5] J. Liu, Y. Yi, A. Proutiere, M. Chiang, and H. V. Poor, "Maximizing utility via random access without message passing," Microsoft Research Technical Report, Tech. Rep., Sept. 2008.

[6] S. Rajagopalan, D. Shah, and J. Shin, "Network adiabatic theorem: an efficient randomized protocol for contention resolution," in *Proc. the 11th ACM Int. Joint Conf. Measurement and Modeling of Computer System (ACM Sigmetric)*, 2009, pp. 133–144.

[7] L. Jiang, D. Shah, J. Shin, and J. Walrand, "Distributed random access algorithm: Scheduling and congestion control," *submitted*, July 2009.

[8] L. Jiang and J. Walrand, "Approaching throughput-optimality in a distributed CSMA algorithm with contention resolution," *Technical Report, UC Berkeley*, March 2009.

[9] A. Eryilmaz, R. Srikant, and J. R. Perkins, "Stable scheduling policies for fading wireless channels," *IEEE/ACM Trans. Networking*, pp. 411–424, Apr. 2005.