Connectivity and Coverage in Failure-Prone Wireless Grid Networks

Technical Report (September 2005)

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***Abstract*— We consider a wireless grid network in which nodes are prone to failure. In the considered failure mode, each node has an independent probability of failure p, and failures are of the crash-stop kind. All nodes are assumed to have a common transmission range r. We establish necessary and sufficient conditions for the minimum transmission range, and hence degree, of each node as a function of the total network size n and the failure probability p, so as to ensure that the network is asymptotically connected with probability 1, as $n \to \infty$. Our results indicate that the degree of each node must be $\Theta(\frac{\ln n}{\ln \frac{1}{p}})$. We also derive conditions for coverage and obtain the same result.**

I. Introduction

We consider the problem of connectivity and coverage in a wireless grid network prone to failure. We show that node degree must be $\Theta(\frac{\ln n}{\ln \frac{1}{p}})$ for asymptotic connectivity and coverage.

II. Network Model

We consider a network model wherein nodes are located on a two-dimensional rectangular toroidal grid (each grid unit is a 1×1 square). The case of a non-toroidal grid will be briefly discussed, and only affects the constant in our results. We designate an origin, and all nodes can be uniquely identified by their grid location (x,y) w.r.t. this origin. All nodes have a common transmission radius r. A message transmitted by a node (x,y) is heard by all nodes within distance r from it (where distance is defined in terms of the particular metric under consideration, and r is assumed to be an integer). The set of these nodes is termed the neighborhood of (x,y).

In this paper, we consider two distance metrics: L_∞ and L_2. The L_∞ metric is the metric induced by the L_∞ norm [1], such that the distance between points (x_1,y_1) and (x_2,y_2) is given by $\max\{|x_1-x_2|,|y_1-y_2|\}$ in the this metric. Thus $nbd(a,b)$ comprises a square of side $2r$ with its centroid at (a,b), and the degree of a node is $4r^2+4r$. The L_2 metric is induced by the L_2 norm [1], and is the Euclidean distance metric. The L_2 distance between points (x_1,y_1) and (x_2,y_2) is given by $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, and $nbd(a,b)$ comprises nodes within a circle of radius r centered at (a,b). The L_∞ metric enables more tractable analysis, from which necessary and sufficient conditions for the L_2 (Euclidean) metric proceed. In Section IX, we further elaborate on how the results for the two metrics are related.

A random failure mode is assumed, wherein each node can fail with probability p independently of other nodes. A failed node simply stops functioning, i.e., failures are of the crash-stop kind.

III. Related Work

Conditions for connectivity and coverage have been formulated in the context of different network models. In [2], it was proved that in a unit area network with uniformly distributed node placement, where nodes have a common transmission radius r, such that $\pi r^2 = \frac{(\log n + c(n))}{n}$, the network is asymptotically connected with probability one iff $c(n) \to \infty$. In [3], an alternate model was considered whereby randomly deployed nodes may modulate their transmission power (and hence range) to ensure that they have a certain

This research is supported in part by the National Science Foundation, and a Vodafone Graduate Fellowship.

number of neighbors. It was proved that each node must be connected to $\Theta(\log n)$ neighbors for asymptotic connectivity with probability one.

A grid network model was considered in [4] where nodes are located at grid locations on a square grid, but may fail independently. Nodes have a common transmission range r. The probability of not failing is specified as p, and it is shown that a sufficient condition for connectivity and coverage is that transmission range r must be set to ensure that node degree is $c_1(\frac{\log n}{p})$ (for some constant c_1). It is also shown that a necessary condition for coverage (and hence for joint coverage and connetivity) is that node degree be at least $c_2(\frac{\log n}{p})$ (for another constant c_2. A fallacy in the above necessary condition was pointed out by [5], and a subsequent correction [6] by the authors of [4] presents examples illustrating that the necessary condition may fail to hold for certain subranges of p. The issue of coverage has been examined in detail in [5] for random, grid, and poisson deployments. However, the necessary and sufficient conditions formulated by them take a more complex form, and do not point to a single $f(n,p)$ such that a degree of $\Theta(f(n,p))$ is both necessary and sufficient for asymptotic coverage. Besides, the necessary condition is formulated for the specific case when $\lim_{n\to\infty} p \to 0$

Our results are closely related to the results of [4]. However, we prove that, given a *failure* probability p, it is necessary and sufficient to have a degree of $\Theta(\frac{\log n}{\log \frac{1}{p}})$ for both connectivity and coverage. Expressed in the notation of [4], we stipulate a degree of $\Theta(\frac{\log n}{\log \frac{1}{1-p}})$. Our results diverge considerably from those of [4] when the failure probability becomes extremely small, and thus our necessary conditions would hold in a certain subdomain where that of [4] would not. However, there is a small sub-domain of p in which our necessary conditions also cease to hold, as with the conditions of [4]. Besides, we work in the L_∞ distance metric, and then map the results to L_2. This yields much simpler proofs. We also remark that our joint sufficient condition for connectivity and coverage is actually sufficient for 9-coverage and not merely 1-coverage (where k-coverage implies that each point is covered by at least k non-faulty nodes).

IV. Notation and Terminology

We briefly describe here notation and terminology that shall be used in this paper. Nodes can identified by their grid location i.e. (x,y) denotes the node at (x,y). The neighborhood of (x,y) comprises all nodes within distance r of (x,y) and is denoted as $nbd(x,y)$. The degree of each node is referred to as d. In L_∞ metric, $d = 4r^2 + 4r$, while the size of a neighborhood (including the neighborhood center) is $d + 1 = 4r^2 + 4r + 1$. The diameter of the network (in terms of distance, and not number of hops) is referred to as D. If n is a perfect square, $D = \sqrt{n}$.

V. Some Useful Mathematical Results

We state some mathematical results that have been used in our proofs:

FACT 1: $\forall x \in [0,1] : \ln \frac{1}{1-x} \geq x$

FACT 2: If $f(n) \leq n^{\frac{1}{2}-\varepsilon}(0 < \varepsilon < \frac{1}{2})$:

$$\lim_{n\to\infty}\left(1 + \frac{f(n)}{n}\right)^n = e^f$$

Proof: See Appendix. ■

FACT 3: If $c > 0$ is a positive constant independent of n, and $b \geq 1$ is another positive constant independent of n, then $\exists n_o \in \mathcal{N}$ such that:
$1 - \frac{1}{(\ln n)^b} \leq \frac{1}{n^{\frac{c}{n}}}$ for $n > n_o$

Proof: See Appendix. ■

VI. Necessary Condition for Connectivity

THEOREM 1: When $p < 1 - \frac{1}{\ln n}$, then in the L_∞ metric, the transmission range r must satisfy $r \geq \max\{1, \Omega(\sqrt{\frac{\ln n}{\ln \frac{1}{p}}})\}$, i.e., the node degree $d \geq \max\{1, \Omega(\frac{\ln n}{\ln \frac{1}{p}})\}$, else $\lim_{n\to\infty} Pr[\text{ disconnection}] = 1$.

Proof: It is obvious that the minimum transmission range required for connectivity is 1, else the degree of all nodes is 0 (except in the case when connectivity loses meaning as all nodes are faulty, and so the network can be deemed connected trivially). Similarly, the network is trivially connected if $r = D$, as all nodes are in direct range of each other. Suppose that $r = \sqrt{\frac{c\ln n}{\ln \frac{1}{p}}}$. Thus, when $p \geq \frac{1}{n^{\frac{c}{n}}}$, $r = \sqrt{\frac{c\ln n}{\ln \frac{1}{p}}} \geq \sqrt{n} \geq D$, and the necessary condition ceases to be relevant (as $r = D$ ensures connectivity).

We show that the network is asymptotically disconnected with probability 1 if $r < \sqrt{\frac{c\ln n}{\ln \frac{1}{p}}}$, for some constant $0 < c < 1$, as long as $p < 1 - \frac{1}{\ln n}$. Note that if $p < 1 - \delta$ for any arbitrarily small constant $\delta > 0$ (independent of n), then for sufficiently large n, the necessary condition would hold for all p. Also note that $1 - \frac{1}{\ln n} < \frac{1}{n^{\frac{c}{n}}}$

for large n (from Fact 3). Thus, the values of p for which our necessary condition holds are those in which the transmission range remains less than D. When $p \geq 1-\frac{1}{n^{1+\varepsilon}}$, all nodes are faulty with probability approaching 1, and the issue of connectivity is moot. When $p \leq \frac{1}{n^c}$, $r=\sqrt{\frac{c\ln n}{\ln\frac{1}{p}}} \leq 1$, and for this range of p, the necessary condition lapses to having the minimum range of 1.

a) $p \leq 1-\frac{1}{\ln n}$: Consider a particular node j in the network. Then, if j is non-faulty, but all its neighbors are faulty, we have a potential disconnection event. Given that there are d neighbors, and each may fail independently with probability p, the probability that j does not fail, but all nodes in $nbd(j)$ fail, is $(1-p)p^d$. We choose a constant $0<c<1$ such that $c\ln n \leq \ln n - 4\ln\ln n$, for sufficiently large n. In general, c can be chosen very close to 1, e.g., $1-\varepsilon(0<\varepsilon<1)$, and the condition will hold for $n>n_o$, for some n_o. Since $p \leq 1-\frac{1}{\ln n}$, we obtain that $\frac{1}{1-p} \leq \ln n$. Let $r \leq \sqrt{\frac{c\ln n}{8\ln\frac{1}{p}}}$. The node degree $d=4r^2+4r \leq 4r^2+4r^2=8r^2$, for $n \geq 1$. Thus, for our choice of r, it turns out that $d \leq c\frac{\ln n}{\ln\frac{1}{p}}$. Then, it may be seen that:

$$\begin{aligned}
&Pr[\text{ A given node } j \text{ is alive, but isolated}] \\
&\quad \geq Pr[j \text{ is alive and all neighbors of } j \text{ are faulty }] \\
&\qquad\qquad = (1-p)p^d > \frac{1}{\ln n} p^{c\frac{\ln n}{\ln\frac{1}{p}}} \\
&\qquad\qquad = \frac{1}{\ln n}\frac{1}{n^c} = \frac{1}{n^c \ln n} \\
&\qquad\qquad \geq \frac{(\ln n)^3}{n} \text{ (from our choice of } c\text{)} \qquad (1)
\end{aligned}$$

Let us mark out a subset of nodes j such that the neighborhoods of these nodes are all disjoint, as in Fig. 1. Then the number of such nodes that we may obtain $= \lfloor \left(\frac{\sqrt{n}}{2r+1}\right)^2 \rfloor \geq \frac{n}{9r^2}-1$ (since $\sqrt{n}$ may not be multiple of $2r+1$). Let I_j be an indicator variable that takes value 1 if j is alive but isolated. Then $Pr[I_j=1] \geq \frac{(\ln n)^3}{n}$, and all I_j's are independent. Let X be a random variable denoting the number of nodes from the chosen set that are alive and isolated. Then $X=\sum I_j$, and $E[X] \geq \frac{(\ln n)^3}{n}\left(\frac{n\ln\frac{1}{p}}{9c\ln n}-1\right) \geq \frac{(\ln n)^3}{n}\frac{n\ln\frac{1}{1-\frac{1}{\ln n}}}{9\ln n} \geq \frac{1}{9}(\ln n)^2\ln\frac{1}{1-\frac{1}{\ln n}} \geq \frac{1}{9}\ln n \to \infty$. We can thus apply the following form of the Chernoff bound [7]:

$$Pr[X \leq (1-\delta)E[X]) \leq \exp(-\frac{\delta^2}{2}E[X]) \qquad (2)$$

Thus, with suitable $0<c<1$ and $\delta=\frac{E[X]-1}{E[X]}$, we obtain

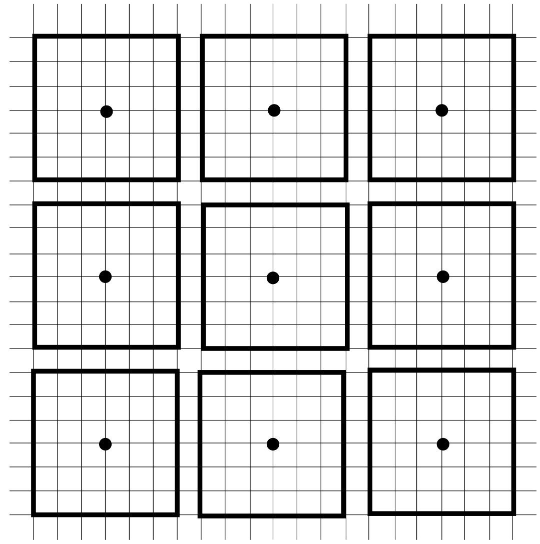

Fig. 1. Nodes having disjoint neighborhoods

that for $p<1-\frac{1}{\ln n}$, if $r \leq \sqrt{\frac{c\ln n}{8\ln\frac{1}{p}}}$, then $E[X]\to\infty$, and hence $\lim_{n\to\infty} Pr[\text{ At least two alive nodes are isolated}]=1$.

Observe that actually the necessary condition would hold for all p such that $E[X]\to\infty$. For instance, the above analysis holds for all $p \leq 1-\frac{1}{(\ln n)^b}$ (where b is a constant), with a corresponding suitably varying choice of c to ensure that $Pr[I_j=1] \geq \frac{(\ln n)^{(b+2)}}{n}$. Besides, if $E[X]\to\gamma>0$, the asymptotic disconnection probability is still a positive finite quantity, and the condition is still necessary for asymptotic connectedness probability to approach 1.

b) $p \geq 1-\frac{1}{n^{1+\varepsilon}}$: When the failure probability becomes so high as to fall in this range, we obtain:

$$\begin{aligned}
&Pr[\text{ Any node is alive}] = 1-p^n \\
&= 1-\left(1-\frac{1}{n^{1+\varepsilon}}\right)^n \to 1-e^{-\frac{1}{n^\varepsilon}} \to 0 \text{ from Fact 2} \qquad (3)
\end{aligned}$$

Thus the network is trivially connected by definition, regardless of degree. ■

VII. Necessary Condition for Coverage

We now show that for the network to be asymptotically covered with probability approaching 1, it is necessary that the transmission range r satisfy: $r \geq \max\{1, \Omega(\sqrt{\frac{\ln n}{\ln\frac{1}{p}}})\}$, i.e., the node degree be $d \geq \max\{1, \Omega(\frac{\ln n}{\ln\frac{1}{p}})\}$.

THEOREM 2: For $p < 1 - \frac{1}{\ln n}$, for a suitable constant $0 < c < 1$, if $d < c\frac{\ln n}{\ln \frac{1}{p}}$:

$$\lim_{n\to\infty} Pr[\text{Some point is not covered}] \to 1$$

Proof: As in the case of connectivity it is obvious that r must be at least 1, else some points will not be covered. We handle two subranges of p separately.

a) $p < 1 - \frac{1}{\ln n}$: The proof relies on subdivision of the network into disjoint neighborhoods, as in Fig. 1. If there exists at least one neighborhood with absolutely no nodes alive (neither the neighborhood center nor its neighbors), then the center of that neighborhood is not covered. Thus we seek to determine the probability of such an event.

We choose a constant $0 < c < 1$ such that $\frac{9}{8}c\ln n \le \ln n - 3\ln\ln n$, for sufficiently large n. This ensures that $\frac{1}{n^c} \ge \frac{(\ln n)^3}{n}$ for large n. Let $r \le \sqrt{\frac{c\ln n}{8\ln\frac{1}{p}}}$. The neighborhood population is given by $d+1 = 4r^2+4r+1 \le 4r^2+4r^2+r^2 = 9r^2$, for $n \ge 1$. Thus, $d+1 \le \frac{9}{8}c\frac{\ln n}{\ln\frac{1}{p}}$. Let I_j be an indicator variable that takes value 1 if there is no alive node in the neighborhood centered at node j, and value 0 otherwise. Then $Pr[X_j = 1] = p^{d+1} = p^{\frac{9}{8}c\frac{\ln n}{\ln\frac{1}{p}}} = \frac{(\ln n)^3}{n}$ (from our choice of c). Let $X = \sum I_j$ be a random variable indicating the number of neighborhoods with no alive node. Then $E[X] = \frac{(\ln n)^3}{9r^2} = \frac{8(\ln n)^2\ln\frac{1}{p}}{9c}$ (after plugging in the chosen value of r). If $p < 1 - \frac{1}{\ln n}$, then $E[X] \ge \ln n(\ln n \ln\frac{1}{1-\frac{1}{\ln n}}) > \ln n \to \infty$ (from Fact 1), and application of the Chernoff bound from Equation 2 yields that $Pr[X=0] \le \exp(-\frac{E[X]}{2}) \to 0$. Thus there is some uncovered region with probability 1.

Similar to the necessary condition for connectivity, observe that this necessary condition would hold for all p such that $E[X] \to \infty$. In particular, the above analysis holds for all $p \le 1 - \frac{1}{(\ln n)^b}$ (where b is a constant), with a corresponding suitably varying choice of c to ensure that $Pr[I_j = 1] \ge \frac{(\ln n)^{(b+2)}}{n}$. Also, if $E[X] \to \gamma > 0$, the asymptotic probability of some point being uncovered is a positive finite quantity, and the condition is still necessary for asymptotic coverage probability to approach 1.

b) $p \ge 1 - \frac{1}{n^{1+\varepsilon}} (0 < \varepsilon < 1)$: We obtain that $Pr[\text{ no nodes alive }] = p^n \ge \left(1 - \frac{1}{n^{1+\varepsilon}}\right)^n$. As $n \to \infty$, the following holds:

$$\lim_{n\to\infty} Pr[\text{some point not covered}] \ge Pr[\text{no node alive}]$$
$$= \lim_{n\to\infty}\left(1 - \frac{1}{n^{1+\varepsilon}}\right)^n \to e^{-\frac{1}{n^{\varepsilon}}} \to 1 \text{ from Fact 2} \quad (4)$$

■

Thus the network is trivially not covered, regardless of transmission range.

VIII. Sufficient Condition for Connectivity and Coverage

We now present a sufficient condition for the asymptotic existence of both connectivity and coverage.

THEOREM 3: When $d > 32\frac{\ln n}{\ln\frac{1}{p}}$, the network is asymptotically connected and covered with probability 1.

Proof:

a) $p \le \frac{1}{n^{1+\varepsilon}}$: When the failure probability is so small as to fall in this range, the probability of even a single node failing approaches 0 asymptotically, and thus connectivity and coverage is trivially ensured even with the minimum transmission range of 1. This may be seen thus:

$$Pr[\text{No failures;full connectivity/coverage}] = (1-p)^n$$
$$\ge \left(1 - \frac{1}{n^{1+\varepsilon}}\right)^n \quad (5)$$

$$\lim_{n\to\infty} Pr[\text{No failures;full connectivity/coverage}]$$
$$\ge \lim_{n\to\infty}\left(1 - \frac{1}{n^{1+\varepsilon}}\right)^n \to e^{-\frac{1}{n^{\varepsilon}}} \to 1 \text{ from Fact 2} \quad (6)$$

b) $p = \Omega(\frac{1}{n})$: Consider the subdivision of the grid as depicted in Fig. 2, so that the resulting cells have x-extents (y-extents) 0 to a, $a+1$ to $a+b$, $a+b+1$ to $2a+b+1$, and so on. Here $a = \lfloor\frac{r}{2}\rfloor$ and $b = r - a = r - \lfloor\frac{r}{2}\rfloor$. Then, each node is within range of all other nodes in the cells adjoining its own. Thus it is obvious that if each square has at least one non-faulty node, there exists a connected backbone that covers all points, and hence all nodes. Thus all non-faulty nodes are connected to each other via this backbone. The dimensions of the cells thus obtained can be $(a+1)^2$, $(a+1)b$ or b^2. Thus the population k of any cell satisfies $k \ge \frac{r^2}{4}$, and the

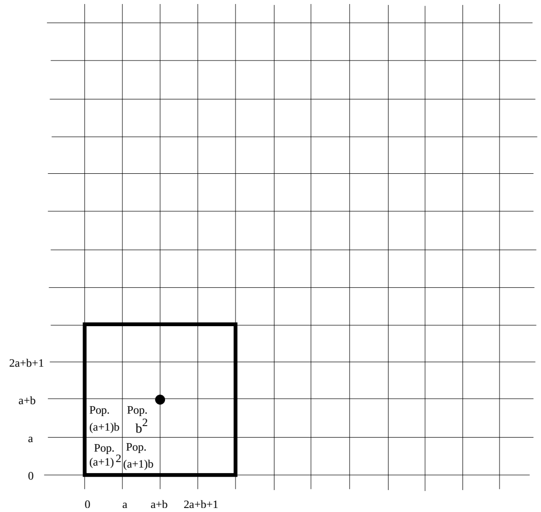

Fig. 2. Subdivision of network into cells

maximum possible number of cells $m \leq \frac{4n}{r^2}$. Then:

$$Pr[\text{ at least one node alive in a given cell }]$$
$$= 1 - p^k \geq 1 - p^{\frac{r^2}{4}}$$
$$\therefore Pr[\text{ at least 1 node alive in each cell}]$$
$$\geq \left(1 - p^{\frac{r^2}{4}}\right)^{\frac{4n}{r^2}} \quad (7)$$

Let us choose $r \geq \sqrt{\frac{8\ln n}{\ln\frac{1}{p}}}$. Then:

$$Pr[\text{at least 1 node alive in each cell}]$$
$$\geq \left(1 - p^{\frac{r^2}{4}}\right)^{\frac{n\ln\frac{1}{p}}{2\ln n}} \quad (8)$$

Since $p \geq \alpha\frac{1}{n}$ for some constant α , $\ln\frac{1}{p} \leq \ln n - \ln\alpha$. Hence:

$$Pr[\text{at least 1 node alive in each cell}]$$
$$\geq \left(1 - p^{\frac{r^2}{4}}\right)^{\frac{n\ln\frac{1}{p}}{2\ln n}} = \left(1 - p^{\frac{2\ln n}{\ln\frac{1}{p}}}\right)^{\frac{n\ln\frac{1}{p}}{2\ln n}} \geq \left(1 - \frac{1}{n^2}\right)^{\frac{n}{2}(1-\frac{\ln\alpha}{\ln n})} \quad (9)$$

Thus, by application of Fact 2, we obtain:

$$\lim_{n\to\infty} Pr[\text{at least 1 node alive in each cell}]$$
$$\geq \lim_{n\to\infty}\left(1 - \frac{1}{n^2}\right)^{\frac{n}{2}(1-\frac{\ln\alpha}{\ln n})}$$
$$\to e^{-\frac{1}{2n}} \to 1 \quad (10)$$

Since this condition ensures connectivity and coverage, we obtain that:

$$\lim_{n\to\infty} Pr[\text{network is connected and covered}] \to 1 \quad (11)$$

■

IX. Conditions in Euclidean Metric

We show that our results derived for L_∞ metric continue to hold for L_2 metric, with only the constants in the theta notation changing.

Lemma 1: If the network is asymptotically connected (covered) in L_∞ for all $r \geq r_{min}$, then the network is connected (covered) asymptotically in L_2 for all $r \geq r_{min}\sqrt{2}$.

Proof: The proof is by contradiction. Suppose that, for a given failure configuration, the network is asymptotically connected in L_∞ for all $r \geq r_{min}$ but is not asymptotically connected for all $r \geq r_{min}\sqrt{2}$ in L_2. Observe that it is possible to circumscribe a L_∞ neighborhood of range r by a L_2 neighborhood of range $r\sqrt{2}$ (Fig. 3). Hence the nodes in an L_2 network of transmission range $r\sqrt{2}$ can be made to simulate the operation of nodes in a L_∞ network with range r (as the L_∞ neigborhood is fully contained within the L_2 neighborhood). This implies that if the L_∞ network of range r is connected (covered), so must be the L_2 network of range $r\sqrt{2}$. If there is some $r \geq r_{min}$ for which the L_∞ network of range r is connected (covered) asymptotically, but the L_2 network of range $r\sqrt{2}$ is not, we obtain a contradiction, as connectedness (coverage) of the L_∞ network would imply connectedness (coverage) of the L_2 network. ■

Lemma 2: If the network is asymptotically disconnected (not covered) in L_∞ for all $r \leq r_{min}$, then the network is disconnected (not covered) asymptotically in L_2 for all $r \leq r_{min}$.

Proof: The proof is by contradiction. Suppose that the network is asymptotically disconnected (not covered) in L_∞ for range r, but is not disconnected (not covered) in L_2 for range r. Observe that an L_∞ neighborhood of transmission range r circumscribes an L_2 neighborhood of range r (Fig. 3). Thus, for any given random failure configuration, if the L_2 network of range r were connected (covered), so would be the L_∞ network of radius r, as we could simply make the nodes in the L_∞ network simulate the behavior of nodes in the L_2 network, and obtain connectedness (coverage). Hence, if the L_2 network of range $r \leq r_{min}$ is not asymptotically

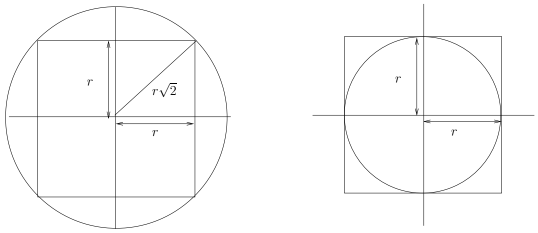

Fig. 3. Relationship between L_∞ and L_2 neighborhoods

disconnected (not covered), the L_∞ network of range $r \leq r_{min}$ must also not be disconnected (not covered). This yields a contradiction. ■

X. Appendix

Proof of FACT 2:

Let $f \leq n^{\frac{1}{2}-\varepsilon}$, where $0 < \varepsilon < \frac{1}{2}$. Let $g(n) = (1+\frac{f}{n})^n$. Then:

$$\ln g = n \ln(1+\frac{f}{n})$$
$$= n\left(\frac{f}{n} - \frac{1}{2}(\frac{f}{n})^2 + \frac{1}{3}(\frac{f}{n})^3 - ....\right)[8]$$
$$= n\sum_{k=1}^{\infty}(-1)^{k-1}\frac{1}{k}(\frac{f}{n})^k = f + \sum_{k=2}^{\infty}(-1)^{k-1}\frac{1}{k}(\frac{f^k}{n^{k-1}}) \to f \text{ as } n \to \infty$$
$$\therefore \lim_{n\to\infty} g(n) = e^f$$

Proof of FACT 3:

$$\because \frac{1}{1-\frac{1}{(\ln n)^b}} \geq e^{\frac{1}{(\ln n)^b}} \text{ (from Fact 1)}$$
$$\therefore 1 - \frac{1}{(\ln n)^b} \leq e^{-\frac{1}{(\ln n)^b}} = \frac{1}{e^{\frac{1}{(\ln n)^b}}} = \frac{1}{e^{\frac{\ln n}{(\ln n)^{(b+1)}}}}$$
$$= \frac{1}{n^{\frac{1}{(\ln n)^{(b+1)}}}} \leq \frac{1}{n^{\frac{c}{n}}} \text{ for large } n$$
$$\because \exists n_o \in \mathcal{N} \text{ s.t. } \frac{1}{(\ln n)^{(b+1)}} \geq \frac{c}{n}, \forall n > n_o$$

References

[1] E. Kreyszig, *Advanced Engineering Mathematics*, 7th ed. John Wiley & Sons, 1993.

[2] P. Gupta and P. R. Kumar, “Critical power for asymptotic connectivity in wireless networks,” in *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming*, W. M. McEneany, G. Yin, and Q. Zhang, Eds. Boston: Birkhauser, 1998, pp. 547–566.

[3] F. Xue and P. R. Kumar, “The number of neighbors needed for connectivity of wireless networks,” *Wirel. Netw.*, vol. 10, no. 2, pp. 169–181, 2004.

[4] S. Shakkottai, R. Srikant, and N. Shroff, “Unreliable sensor grids: Coverage, connectivity, and diameter,” in *Proc. of Infocom 2003*, 2003.

[5] S. Kumar, T. H. Lai, and J. Balogh, “On k-coverage in a mostly sleeping sensor network,” in *MobiCom ’04: Proceedings of the 10th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM Press, 2004, pp. 144–158.

[6] S. Shakkottai, R. Srikant, and N. Shroff, “Correction to unreliable sensor grids: Coverage, connectivity, and diameter,” Personal Communication, 2005.

[7] R. Motwani and P. Raghavan, *Randomized algorithms*. Cambridge University Press, 1995.

[8] G. B. Thomas, Jr. and R. L. Finney, *Calculus and Analytic Geometry*. Addison-Wesley Publishing Company, 1992.