Capacity of Multichannel Wireless Networks: Impact of Channels, Interfaces, and Interface Switching Delay*

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Abstract

This paper studies how the capacity of a static multichannel network scales as the number of nodes, n, increases. Gupta and Kumar have determined the capacity of single channel networks, and those bounds are applicable to multichannel networks as well, provided each node in the network has a dedicated interface per channel. In this work, we establish the capacity of general multichannel networks wherein the number of interfaces, m, may be smaller than the number of channels, c. We show that the capacity of multichannel networks exhibits different bounds that are dependent on the ratio between c and m. When the number of interfaces per node is smaller than the number of channels, there is a degradation in the network capacity in many scenarios. However, one important exception is a random network with up to $O(\log n)$ channels, wherein the network capacity remains at the Gupta and Kumar bound of $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec, independent of the number of interfaces available at each node. Since in many practical networks, number of channels available is small (e.g., IEEE 802.11 networks), this bound is of practical interest. This implies that it may be possible to build capacity-optimal multichannel networks with as few as one interface per node. We also extend our model to consider the impact of interface switching delay, and show that in a random network with up to $O(\log n)$ channels, switching delay may not affect capacity if multiple interfaces are used.

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I. Introduction

In this report, we study the asymptotic capacity of multichannel wireless networks with varying number of interfaces. Past research on wireless network capacity [2], [3] has typically considered wireless networks with a single channel, although the results are applicable to a wireless network with multiple channels as well, provided that at each node there is a dedicated interface per channel. With a dedicated interface per channel, a node can use all the available channels simultaneously. However, the number of available channels in a wireless network can be fairly large, and it may not be feasible to have a dedicated interface per channel at each node. When nodes are not equipped with a dedicated interface per channel, then *capacity degradation* may occur, compared to using a dedicated interface per channel.

In this report, we characterize the impact of number of channels and interfaces per node on the network capacity, and show that in certain scenarios, even with only a single interface per node, there is no capacity degradation. This implies that it may be possible to build capacity-optimal multichannel networks with as few as one interface per node. Our initial analysis assumes that the interface switching delay is zero, which may not be valid in practice. Nevertheless, even when interface switching delay is accounted for, capacity-optimal performance can be achieved by using only a few interfaces per node. In addition, if each node has a single interface that is never switched, then there is a degradation in the network capacity. However, with only two interfaces per node, there is no capacity degradation even if the interfaces are not switched.

The rest of this report is organized as follows. We present the channel and network model, as well as an overview of the main results in Section II. We present related work in Section III. In Section IV, we establish the capacity of multichannel networks under arbitrary network setting. Section V establishes the capacity of multichannel networks under random network setting. Section VI characterizes the impact of interface switching delay, and Section VII studies the capacity when interfaces do not switch at all. We conclude this report in Section VIII.

II. Preliminaries

In this section, we first define the channel and network model, and then provide an overview of results.

A. Channel and interface model

We consider a static wireless network containing n nodes. In our model there are c channels, and we assume that every node is equipped with m interfaces, $1 \leq m \leq c$. We assume that an interface is capable of transmitting or receiving data on any one channel at a given time. We use the notation (m, c)-network to refer to a network with m interfaces per node, and c channels.

We define two channel models to represent the data rate supported by each channel:

Channel Model 1: In model 1, we assume that the total data rate possible by using all channels is W. The total data rate is divided equally among the channels, and therefore the data rate supported by any one of the c channels is W/c. This was the channel model used by Gupta and Kumar [2], and we primarily use this model in our analysis. In this model, as the number of channels increases, each channel supports a smaller data rate. This model is applicable to the scenario where the total available bandwidth is fixed, and new channels are created by partitioning existing channels.

Channel Model 2: In model 2, we assume that each channel can support a fixed data rate of W, independent of the number of channels. Therefore, the aggregate data rate possible by using all c channels is Wc. This model is applicable to the scenario where new channels are created by utilizing additional frequency spectrum.

The capacity results are derived using channel model 1. However, all the derivations are applicable for channel model 2 as well, and the results for model 2 are obtained by replacing W in the results of model 1 by Wc.

B. Network and traffic model

We study the capacity of static multichannel wireless networks under the two settings introduced by Gupta and Kumar [2].

Arbitrary Networks: In the arbitrary network setting, the location of nodes, and traffic patterns can be controlled. Since any suitable traffic pattern and node placement can be used, the bounds for this scenario are applicable to any network. Therefore, the arbitrary network bounds may be viewed as the *best case* bounds on network capacity, as the bounds are applicable to all networks. The network capacity is measured in terms of "bit-meters/sec" (originally introduced by Gupta and Kumar [2]). The network is said to transport one "bit-meter/sec" when one bit has been transported across a distance of one meter in one second.

Random Networks: In the random network setting, node locations are randomly chosen, i.e. independently and uniformly chosen, on the surface of an unit torus. Each node sets up one flow to a randomly chosen destination[1]. The network capacity is defined to be the aggregate throughput over all the flows in the network, and is measured in terms of bits/sec.

C. Definitions

We use the following notation [4] to represent asymptotic bounds:

1) $f(n) = O(g(n))$ implies that there exists some constant d and integer N such that $f(n) \leq dg(n)$ for $n > N$.

[1]Gupta and Kumar [2] choose a random point and then choose the node nearest to the chosen point as the destination. Our model is slightly different as we directly choose a random node as the destination.

2) $f(n) = o(g(n))$ implies that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.
3) $f(n) = \Omega(g(n))$ implies $g(n) = O(f(n))$.
4) $f(n) = \omega(g(n))$ implies $g(n) = o(f(n))$.
5) $f(n) = \Theta(g(n))$ implies $f(n) = O(g(n))$ and $g(n) = O(f(n))$.
6) $MIN_O\,(f(n), g(n))$ is equal to $f(n)$, if $f(n) = O(g(n))$, else, is equal to $g(n)$.

The bounds for random networks hold *with high probability (whp)*. In this report, *whp* implies "with probability 1 when $n \to \infty$."

D. Main Results

Gupta and Kumar [2] have shown that in an arbitrary network, network capacity scales as $\Theta\left(W\sqrt{n}\right)$ bit-meters/sec, and in a random network, the network capacity scales as $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec. Under the channel model 1, which was the model used by Gupta and Kumar [2], the capacity of a network with a single channel and one interface per node (that is, a $(1,1)$-network in our notation) is equal to the capacity of a network with c channels and $m = c$ interfaces per node (that is, a (c,c)-network). This equivalence arises because the c interfaces can operate in parallel over channels of data rate $\frac{W}{c}$ to mimic the operation of one interface operating over a channel of data rate W (this is formally proved in Lemma 1). Furthermore, under both channel models, the capacity of a (c,c)-network is at least as large as the capacity of a (m,c)-network, when $m \le c$ (this is trivially true, by not using $c - m$ interfaces in the (c,c)-network).

In the results presented in this report, we capture the impact of using fewer than c interfaces per node by establishing the *loss in capacity*, if any, of a (m,c)-network in comparison to a (c,c)-network. We show that there are distinct capacity regions, the boundaries of which depend on the ratio $\frac{c}{m}$, and not on the exact values of either c or m. Here we present an overview of the main results, under channel model 1.

1. Results for arbitrary network: The network capacity of a (m,c)-network has two regions (see Figure 1) as follows (from Theorem 2 and Theorem 4):

1) When $\frac{c}{m}$ is $O(n)$, the network capacity is $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec (segment A-B in Figure 1). Compared to a (c,c)-network, there is a capacity loss by a factor of $1 - \sqrt{\frac{m}{c}}$.
2) When $\frac{c}{m}$ is $\Omega(n)$, the network capacity is $\Theta\left(W\frac{nm}{c}\right)$ bit-meters/sec (line B-C in Figure 1). In this case, there is a larger capacity degradation than case 1, as $\frac{nm}{c} \le \sqrt{\frac{nm}{c}}$ when $\frac{c}{m} \ge n$.

Therefore, there is always a capacity loss in arbitrary networks whenever the number of interfaces per node is fewer than the number of channels.

2. Results for random network: The network capacity of a (m,c)-network has three regions (see Figure 2) as follows (from Theorem 6 and Theorem 9):

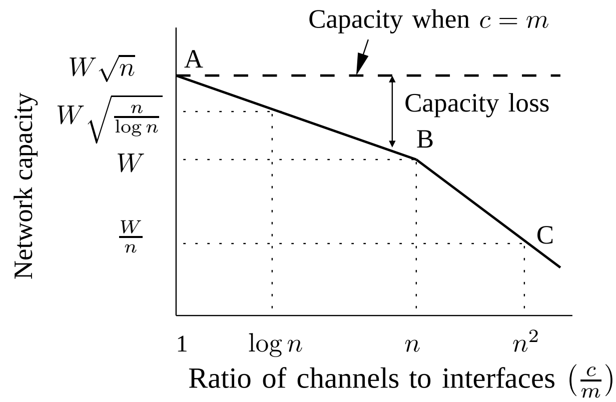

Fig. 1. Impact of number of channels on capacity scaling in arbitrary networks (figure is not to scale).

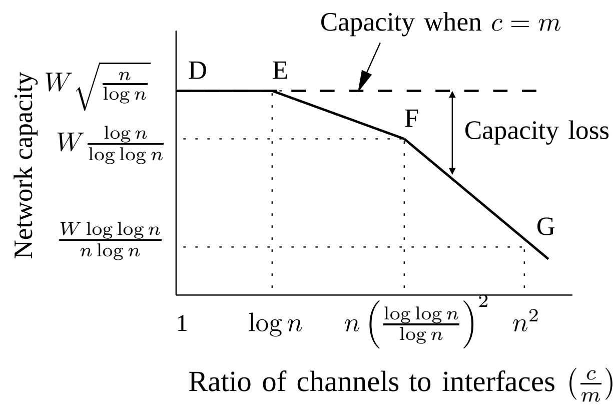

Fig. 2. Impact of number of channels on capacity scaling in random networks (figure is not to scale).

1) When $\frac{c}{m}$ is $O(\log n)$, network capacity is $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec (segment D-E in Figure 2). In this case, *there is no loss* compared to a (c, c)-network. Hence, in many practical scenarios where c may be constant or small, *a single interface per node suffices.*
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec (segment E-F in Figure 2). In this case, there is some capacity loss. Furthermore, in this region, the capacity of a (m, c)-random network *is the same* as that of a (m, c)-arbitrary network (segment E-F in Figure 2 overlaps part of segment A-B in Figure 1), implying that "randomness" does not incur a capacity penalty.
3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $\Theta\left(\frac{Wnm\log\log n}{c\log n}\right)$ bits/sec (line F-G in Figure 2). In this case, there is a larger capacity degradation than case 2. Furthermore, in this region, the capacity of a (m, c)-random network *is smaller than* that of a (m, c)-arbitrary network, in contrast to case 2.

3. Impact of switching delay: The results presented above are derived under the assumption that there is no delay in switching an interface from one channel to another. However, we show that in a random network with up to $O(\log n)$ channels, even if interface switching delay is considered, the network capacity is not reduced, provided a few additional interfaces are provisioned for at each node. This implies that it may be possible to hide the interface switching delay by using extra interfaces in conjunction with carefully designed routing and transmission scheduling protocols.

4. Impact of keeping interfaces fixed: In practice, protocol implementation can be simplified if interfaces are fixed to channels. We show that if every node has a single interface, and the interface is never switched (after initially assigning the interface to some channel), then there is a loss in the network capacity. This loss in capacity can be avoided, by having only two interfaces per node, even if the interfaces do not switch.

III. Related work

In their seminal work, Gupta and Kumar [2] established the capacity of ad hoc wireless networks. The results are applicable to single channel wireless networks, or multichannel wireless networks where every node has a dedicated interface per channel. We extend the results of Gupta and Kumar to those multichannel wireless networks where nodes may not have a dedicated interface per channel, and we also consider the impact of interface switching delay on network capacity.

Grossglauser and Tse [3] showed that mobility can improve network capacity, though at the cost of increased end-to-end delay. Subsequently, other research [5], [6] has analyzed the trade-off between delay and capacity in mobile networks. Gamal et al. [4] characterize the optimal throughput-delay trade-off for both static and mobile networks. In this thesis, we adapt some of the proof techniques presented by Gamal et al. [4] to the multichannel capacity problem. Lin et al. [7], [8] also study the throughput-delay trade-off in wireless networks.

Recent results have shown that the capacity of wireless networks can be enhanced by introducing infrastructure support [9]–[11]. Other approaches for improving network capacity include the use of directional antennas [12], and the use of unlimited bandwidth resources (UWB), albeit with power constraints [13], [14]. Li et al. [15] have used simulations to evaluate the capacity of multichannel networks based on IEEE 802.11. Other research on capacity is based on considerations of alternate communication models [16]–[18], but do not consider the multichannel scaling problem.

Kodialam et al. [19] have studied the throughput achievable in a multichannel multi-interface network by using constrained optimization techniques. Their work is applicable to scenarios where the network topology and traffic patterns are known a priori. Alicherry et al. [20] have considered a similar multichannel multi-interface problem, but for a restricted class of mesh networks (where all traffic is directed toward gateway nodes). Zhang et al. [21] have studied the benefits of jointly optimizing both routing and scheduling in multichannel multi-interface networks. All these works are well suited for network planning, but are less useful in understanding scaling properties of the network.

IV. CAPACITY RESULTS FOR ARBITRARY NETWORKS

We assume that all nodes transmit at the same data rate, and use the same transmission power. We model the impact of interference by using the protocol model proposed by Gupta and Kumar [2]. The transmission from a node i to a node j on some channel x is successful, if for every other node k simultaneously transmitting on channel x, the following condition holds:

$$d(k,j) \geq (1+\Delta)d(i,j), \;\; \Delta > 0$$

where $d(i,j)$ is the distance between nodes i and j, and Δ is a "guard" parameter to ensure that any other concurrently transmitting nodes are sufficiently farther away from the receiver to prevent excessive interference.

It is shown in [2] that the protocol model is equivalent to an alternate physical model that is based on received Signal-to-Interference-Noise Ratio (SINR) (when path loss exponent is greater than 2). Therefore, the results in this thesis are applicable under the physical model as well. We do not consider other physical layer characteristics such as channel fading in our analysis. We derive the capacity results for arbitrary and random networks under the assumption that there is no switching delay. We extend our model to consider the impact of switching delay in Section VI.

In an arbitrary network, the location of nodes, and traffic patterns can be controlled. Recall that the network is said to transport one "bit-meter/sec" when one bit has been transported across a distance of one meter in a second. The network capacity of an arbitrary network is measured in terms of bit-meters per second, instead of bits per second. The bit-meters/sec metric is a measure of the "work" that is done by the network in transporting bits. In the case of random networks, the average distance traveled by any bit is $\Theta(1)$, and therefore the "bit-meters/sec" and "bits/sec" capacity is of the same order.

We assume that n nodes can be located anywhere on the surface of a torus of unit area, as in [4]. The assumption of a torus enables us to avoid technicalities arising out of edge effects, but the results are applicable for nodes located on an unit square as well. We first establish an upper bound on the network capacity of arbitrary networks, and then construct a network to prove that the bound is tight.

A. Upper bound on capacity

The capacity of multichannel arbitrary networks is limited by two constraints (described below), and each of them is used to obtain a bound on the network capacity. The minimum of the two bounds (the bounds depend on ratio between the number of channels c and the number of interfaces m) is an upper bound on the network capacity. While there may be other constraints on capacity as well, the constraints we consider are sufficient to provide a tight bound. Later in this section, we will present a lower bound that matches the upper bound established by the two constraints, which validates our claim that the constraints are tight. We derive the bounds under channel model 1, although the derivation can be applied to channel model 2 as well[2].

Constraint 1 – Interference constraint: The capacity of any wireless network is constrained by interference. Since the wireless channel is a shared medium, under the assumed protocol model of interference, two nodes simultaneously receiving a packet from two different transmitters must have a minimum separation between them, which depends on Δ. This implies that there is a bound on the maximum number of simultaneous transmissions in the network. Based on this observation, using the proof techniques presented in [2] with some modifications to account for multiple interfaces and channels, one bound on the network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec. The detailed derivation is below in Theorem 1.

Theorem 1: An upper bound on the capacity of a (m, c)-network under the arbitrary network setting is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec under channel model 1.

Proof: We prove the result under channel model 1. The proof is based on a proof in [2]. We assume that nodes are synchronized, and slotted transmissions of duration τ are used. We assume that each source node originates λ bits/sec. Let the average distance between source and destination pairs be $\bar{L}$. Therefore, the capacity of the network is $\lambda n\bar{L}$ bit-meters/sec.

We consider any time period of length one second. In this time interval, consider a bit b, $1 \le b \le \lambda n$. We assume that bit b traverses $h(b)$ hops on the path from its source to its destination, where the h-th hop traverses a distance of r_b^h. Since the distance traversed by a bit from its source to its destination is at least equal to the length of the line joining the source and the destination, by summing over all bits we obtain,

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} r_b^h \ge \lambda n\bar{L} \tag{1}$$

Let us define H to be the total number of hops traversed by all transmitted bits in a second, i.e., $H = \sum_{b=1}^{\lambda n} h(b)$. Therefore, the number of bits transmitted by all nodes in a second (including bits relayed) is equal to H. Since each node has m interfaces, and each interface transmits over a channel with rate W/c (assuming channel model 1), the total bits that can be transmitted by all nodes over all interfaces is at most $\frac{Wmn}{2c}$ (Transporting a bit across one hop requires *two* interfaces, one each at the transmitting and the receiving nodes). Hence, we have,

$$H \le \frac{Wmn}{2c} \tag{2}$$

Under the protocol model, a transmission over a hop of length r is successful only if there is no other node transmitting within a distance of $(1 + \Delta)r$ of the receiver. Suppose node A is transmitting a bit to node B, while node C is simultaneously transmitting a bit to node D, and both the transmissions are over a common channel. Then, using the protocol interference model, both transmissions are successful only if

[2]Recall that the results under channel model 2 can be obtained by replacing W with Wc in the results derived under channel model 1.

$$d(C,B) \geq (1+\Delta)d(A,B)$$

$$d(A,D) \geq (1+\Delta)d(C,D)$$

Adding the above two expressions together, and applying triangle inequality, we obtain,

$$d(B,D) \geq \frac{\Delta}{2}(d(A,B)+d(C,D))$$

This implies that the receivers of two simultaneous transmissions have to be separated by a distance proportional to the distance from their senders. This may be viewed as each hop consuming a disk of radius $\frac{\Delta}{2}$ times the length of the hop around each receiver. Since the area "consumed" on each channel is bounded above by the area of the domain (1 sq meter), summing over all channels (which can in total potentially transport W bits) we have the constraint,

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{\pi\Delta^2}{4}(r_b^h)^2 \leq W \tag{3}$$

which can be rewritten as,

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{4W}{\pi\Delta^2 H} \tag{4}$$

Since the expression on the left hand side is convex, we have,

$$(\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H}r_b^h)^2 \leq \sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \tag{5}$$

Therefore, from (4) and (5),

$$\sum_{b=1}^{\lambda n}\sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4WH}{\pi\Delta^2}} \tag{6}$$

Substituting for H from (2), and using (1) we have,

$$\lambda n\bar{L} \leq W\sqrt{\frac{2mn}{\pi\Delta^2 c}} \tag{7}$$

This proves that the network capacity of an arbitrary network is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec under channel model 1. ■

Constraint 2 – Interface bottleneck constraint: The capacity of a wireless network is also constrained by the maximum number of bits that can be transmitted simultaneously over all interfaces in the network. Since each node has m interfaces, there are a total of mn interfaces in the (m,c)-network. Each interface can transmit at a rate of $\frac{W}{c}$ bits/sec. Also, the maximum distance a bit can travel in the network is $O(1)$ meters. Hence, the total network capacity is at most $O\left(W\frac{nm}{c}\right)$ bit-meters/sec. This bound is tight when $\frac{c}{m}$ is $\Omega(n)$.

Combining the two constraints, the network capacity is $O\left(MIN_O\left(W\sqrt{\frac{nm}{c}}, W\frac{nm}{c}\right)\right)$ bit-meters/sec, under channel model 1. Therefore, we have the following theorem on the network capacity of arbitrary networks (Figure 1 has a pictorial representation).

Theorem 2: The upper bound on the capacity of a (m, c)-arbitrary network under channel model 1 is as follows:

1) When $\frac{c}{m}$ is $O(n)$, network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec.
2) When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $O\left(W\frac{nm}{c}\right)$ bit-meters/sec.

The result for channel model 2 can be similarly derived, and is given by:

Theorem 3: The upper bound on the capacity of a (m, c)-arbitrary network under channel model 2 is as follows:

1) When $\frac{c}{m}$ is $O(n)$, network capacity is $O\left(W\sqrt{nmc}\right)$ bit-meters/sec.
2) When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $O\left(Wnm\right)$ bit-meters/sec.

The network capacity of a (c, c)-network is $O\left(W\sqrt{n}\right)$ bit-meters/sec under channel model 1, which was the result obtained by Gupta and Kumar [2]. When fewer interfaces are available, there is a capacity degradation by at least a factor of $1-\sqrt{\frac{m}{c}}$. Intuitively, the capacity degradation arises because the total bits that can be simultaneously transmitted decreases.

B. Constructive lower bound

In this section, we construct a network to establish a lower bound on the network capacity. The lower bound matches the upper bound, implying that the bounds are tight. We first establish two results that we use in the rest of the report. The results are proved under channel model 1, but hold for channel model 2 as well.

Lemma 1: Suppose m, c, $\tilde{c}$ are positive integers such that $\tilde{c} = \frac{c}{m}$. Then, a (m, c)-network can support at least the capacity supported by a $(1, \tilde{c})$-network, under channel model 1.

Proof: Consider a (m, c)-network. We group the c channels into $\tilde{c}$ groups (numbered from 1 to $\tilde{c}$), with m channels per group as shown in Figure 3. Specifically, channel group $i,\ 1 \leq i \leq \tilde{c}$, contains all channels j such that $(i-1)m + 1 \leq j \leq im$.

Assume that time on the channels is divided into slots of duration τ. Consider any slot s. Suppose a node X in the $(1, \tilde{c})$-network has its interface on some channel i, $1 \leq i \leq \tilde{c}$, in slot s. We simulate this behavior in the (m, c)-network by assigning the m interfaces of X in the slot s to the m channels in the channel group i. In this fashion, in any slot, the m interfaces of any node in the (m, c)-network are mapped to a channel group. The aggregate data rate of each channel group is $Wm/c = W/\tilde{c}$ (since $c = m\tilde{c}$). Therefore, a channel group in the (m, c)-network can support the same data rate as a channel in the $(1, \tilde{c})$-network. This mapping allows the (m, c)-network to mimic the behavior of $(1, \tilde{c})$-network; the $W\tau/\tilde{c}$ bits sent on some channel in any time slot s in the $(1, \tilde{c})$-network can be simulated by sending $W\tau/c$ bits (in the same slot s) on each of the m channels in the corresponding channel group of the (m, c)-network. Hence, a (m, c)-network can support the capacity of a $(1, \tilde{c})$ network, when $c = m\tilde{c}$.

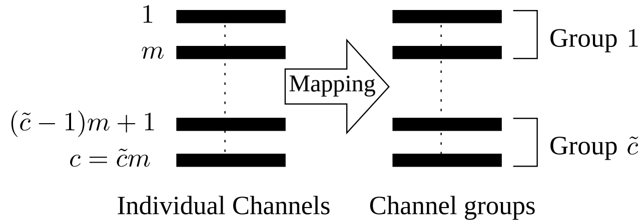

Fig. 3. Lemma 1 construction: Forming $\tilde{c}$ channel groups, with m channels per group, in a (m, c)-network.

■

Lemma 2: Suppose m and c are positive integers. Then, a (m, c)-network can support at least $\frac{1}{2}$ the capacity supported by a $(1, \lfloor \frac{c}{m} \rfloor)$-network, under channel model 1.

Proof: Suppose $\lfloor \frac{c}{m} \rfloor = \frac{c}{m}$. Then the result directly follows from the previous lemma. Otherwise, $m < c$, and we use $c' = m \lfloor \frac{c}{m} \rfloor$ of the channels in the (m, c)-network, and ignore the rest of the channels. This can be viewed as a (m, c')-network, with a total data rate of $W' = W \frac{m}{c} \lfloor \frac{c}{m} \rfloor$ (as each channel supports $\frac{W}{c}$ bits/sec). Using Lemma 1, a (m, c')-network with total data rate of W' can support at least the capacity of a $(1, \lfloor \frac{c}{m} \rfloor)$-network with total data rate of W'. However, when $W' < W$, the (m, c')-network with total data rate W' can achieve only a fraction $\frac{W'}{W}$ of the capacity of a $(1, \lfloor \frac{c}{m} \rfloor)$-network with total data rate W (instead of W'). Now,

$$\begin{aligned}
\frac{W'}{W} &= \frac{m}{c} \left\lfloor \frac{c}{m} \right\rfloor \\
&= \frac{\lfloor \frac{c}{m} \rfloor}{\frac{c}{m}} \\
&\geq \frac{\lfloor \frac{c}{m} \rfloor}{\lfloor \frac{c}{m} \rfloor + 1}, \text{ since } \frac{c}{m} \leq \left\lfloor \frac{c}{m} \right\rfloor + 1 \\
&\geq \frac{1}{2}, \text{ since } \left\lfloor \frac{c}{m} \right\rfloor \geq 1
\end{aligned}$$

Hence, a (m, c)-network can support at least $\frac{1}{2}$ the capacity supported by a $(1, \lfloor \frac{c}{m} \rfloor)$ network. Hence, asymptotically, a (m, c)-network has the same order of capacity as a $(1, \lfloor \frac{c}{m} \rfloor)$-network. ■

We now provide the construction to establish that a capacity of $\Omega\left(MIN_O\left(W\sqrt{\frac{nm}{c}}, W\frac{nm}{c}\right)\right)$ bit-meters/sec is achievable in a $(1, c)$-network, under the channel model 1. The result is then extended to a (m, c)-network by using Lemma 2.

Step 1: We consider a torus of unit area. Let $k = \min\left(c, \frac{n}{8}\right)$. This implies that $k \leq c$. Partition the square area into $\frac{n}{8k}$ equal-sized square cells, and place $8k$ nodes in each cell. Since the total area is 1, each cell has an area of $\frac{8k}{n}$, and sides of length $l = \sqrt{\frac{8k}{n}}$.

Step 2: The $8k$ nodes within each cell are distributed by placing k nodes at each of the eight positions shown in Figure 4. Nodes placed at locations S1, S2, S3, S4 act as senders, and nodes placed at remaining locations act

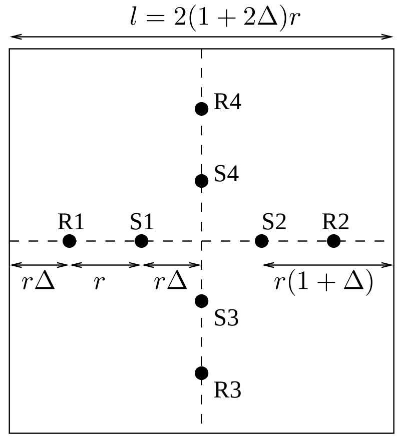

Fig. 4. The placement of nodes within a cell. There are k nodes at each of the labeled positions.

as receivers. The sender locations S1 through S4 are at a distance of $r\Delta$ from the center of the cell (recall that Δ is the "guard" parameter from the protocol model of interference), where $r = \frac{l}{2(1+2\Delta)} = \frac{1}{(1+2\Delta)}\sqrt{\frac{2k}{n}}$. The receiver locations R1 through R4 are at a distance of $r(1 + \Delta)$ from the center of the cell. Therefore, the distance between S1-R1, S2-R2, S3-R3, and S4-R4 is equal to r. Each receiver location is at a distance of $r\Delta$ from nearest edge of the cell, and each sender location is at a distance of $r(1 + \Delta)$ from the nearest edge of the cell.

Step 3: Label the k nodes in any location (S1 through S4, R1 through R4) as 1 through k. The j^{th} node in each sender location, $1 \leq j \leq k$, communicates with the j^{th} node in the nearest receiver location (at a distance of r) on channel j. Consider any pair of communicating nodes A and B that are located at, say, S1 and R1 respectively. Then, the nearest senders within the cell, other than A (located at S1), which are sending on the same channel as A are located at one of S2, S3, S4, and are at least a distance of $r(1+\Delta)$ away from B (located at R1). Similarly, in every cell, senders are at least $r(1+\Delta)$ distance from the cell boundary. Therefore, senders in adjacent cells of B are at least a distance of $r(1 + \Delta)$ away from B as well. Hence, under the protocol model of interference, the transmission between A and B is not interfered with by any other transmission in the network, and this property holds for all communicating pairs.

From the above construction, there are $\frac{n}{2}$ pairs of nodes in the $(1, c)$-network, each transmitting at a rate of $\frac{W}{c}$ over a distance $r = \frac{1}{(1+2\Delta)}\sqrt{\frac{2k}{n}}$. Hence, the total capacity of the network (summing over all n nodes) is $\frac{n}{2}\frac{W}{c}r = \frac{W}{c}\frac{1}{(1+2\Delta)}\sqrt{\frac{nk}{2}}$ bit-meters/sec. Recall that $k = \min\left(c, \frac{n}{8}\right)$. Substituting for k in the above derivation, we obtain the capacity of a $(1, c)$-network to be $\Omega\left(MIN_O\left(W\sqrt{\frac{n}{c}}, W\frac{n}{c}\right)\right)$ bit-meters/sec under channel model 1, since Δ is a constant.

Using Lemma 2, the capacity of a (m, c)-network, under the arbitrary network setting and channel model 1, is $\Omega\left(MIN_O\left(W\sqrt{\frac{n}{\lceil\frac{c}{m}\rceil}}, W\frac{n}{\lceil\frac{c}{m}\rceil}\right)\right)$ bit-meters/sec. Since $\frac{1}{\lceil\frac{c}{m}\rceil} \geq \frac{1}{\frac{c}{m}}$, we have the capacity of arbitrary networks to be

$\Omega\left(MIN_O\left(W\sqrt{\frac{mn}{c}}, W\frac{mn}{c}\right)\right)$ bit-meters/sec, which leads to the following theorem:

Theorem 4: The achievable network capacity of a (m, c)-arbitrary network under channel model 1 is as follows:

1) When $\frac{c}{m}$ is $O(n)$, network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec.
2) When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $\Omega\left(W\frac{nm}{c}\right)$ bit-meters/sec.

The result for channel model 2 can be similarly derived, and is given by:

Theorem 5: The achievable network capacity of a (m, c)-arbitrary network under channel model 2 is as follows:

1) When $\frac{c}{m}$ is $O(n)$, network capacity is $\Omega\left(W\sqrt{nmc}\right)$ bit-meters/sec.
2) When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $\Omega\left(Wnm\right)$ bit-meters/sec.

The upper bound and lower bound of the capacity of arbitrary networks have the same order, indicating that the bounds are tight.

C. Implications

A common scenario is when the number of channels is not too large ($\frac{c}{m} = O(n)$). Under this scenario, the capacity of a (m, c)-network in the arbitrary setting scales as $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ under channel model 1. Similarly, under channel model 2, the capacity of the network scales as $\Theta\left(W\sqrt{nmc}\right)$. Under either model, the capacity of a (m, c)-network goes down by a factor of $1 - \sqrt{\frac{m}{c}}$, when compared with a (c, c)-network. Therefore, doubling the number of interfaces at each node (as long as number of interfaces is smaller than the number of channels) increases the channel capacity by a factor of only $\sqrt{2}$. Furthermore, the ratio between m and c demarcates the capacity regions, rather than the individual values of m and c. Increasing the number of interfaces may result in a linear increase in the cost but only a sub-linear (proportional to square-root of number of interfaces) increase in the capacity. Therefore, the optimal number of interfaces to use may be smaller than the number of channels depending on the relationship between cost of interfaces and utility obtained by higher capacity.

Different network architectures have been proposed for utilizing multiple channels when the number of available interfaces is smaller than the number of available channels [22]–[24]. The construction used in proving lower bound shows that capacity is maximized when all channels are utilized. One architecture used in the past [22] is to use only m channels when m interfaces are available, leading to wastage of the remaining $c - m$ channels. That architecture results in a factor of $1 - \frac{m}{c}$ loss in capacity which can be significantly higher than the optimal $1 - \sqrt{\frac{m}{c}}$ loss (when $\frac{c}{m} = O(n)$). Hence, in general, higher capacity may be achievable by *architectures that use all channels,* possibly by dynamically switching channels.

V. Capacity results for random networks

We assume that n nodes are randomly located on the surface of a torus of unit area. Each node selects a destination uniformly at random from the remaining nodes[3], and sends $\lambda(n)$ bits/sec to the destination. The highest value of $\lambda(n)$ which can be supported by *every* source-destination pair with high probability is defined as the *per-node throughput* of the network. The traffic between a source-destination pair is referred to as a "flow". Since there are a total of n flows, the network capacity is defined to be $n\lambda(n)$.

Note that each node picks a destination node randomly, and therefore, a node may be the destination of multiple flows. Let $D(n)$ be the maximum number of flows for which a node in the network is a destination. We use the following result to bound $D(n)$.

Lemma 3: The maximum number of flows for which a node in the network is a destination, $D(n)$, is $\Theta\left(\frac{\log n}{\log \log n}\right)$, with high probability.

Proof: The process of nodes selecting a random destination may be mapped to the well-known "Balls into Bins" problem [25]. Each source node may be viewed as a "ball", and each destination node may be viewed as a "bin". The process of selecting a destination node may be viewed as randomly dropping a "ball" into a "bin". Based on this mapping, the proof of the lemma follows from well-known results (cf. [25], Section 4). ■

A. Upper bound for random networks

The capacity of multichannel random networks is limited by three constraints, and each of them is used to obtain a bound on the network capacity. The minimum of the three bounds (the bounds depend on ratio between the number of channels c and the number of interfaces m) is an upper bound on the network capacity. While there may be other constraints on capacity as well, the constraints we consider are sufficient to provide a tight bound. We derive the bounds under channel model 1, but the results are applicable under channel model 2 as well.

Constraint 1 – Connectivity constraint: The capacity of random networks is constrained by the need to ensure that the network is connected, so that every source-destination pair can successfully communicate. Since node locations are randomly chosen, there is some minimum transmission range each node should use to ensure that the network is connected. Since all transmissions cover at least an area proportional to the square of the minimum transmission range, there is a bound on the number of simultaneous transmissions that can occur in the network. Based on this observation, Gupta and Kumar [2] have presented one bound on the network capacity to be $O\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec. This bound is applicable to multichannel networks as well.

[3]Recall that Gupta and Kumar [2] choose a random point and then choose the node nearest to the chosen point as the destination. Our model is slightly different as we directly choose a random node as the destination.

Constraint 2 – Interference constraint: A random network is a special case of an arbitrary network, and therefore the arbitrary network constraints are applicable to random networks as well. Therefore, the capacity of multichannel random networks is also constrained by interference (this is same as the constraint 1 listed for arbitrary networks in Section IV-A). This constraint was already captured in the upper bound for arbitrary networks, and we had obtained a bound of $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec. In a random network, each of the n source-destination pairs are separated by an average distance of $\Theta(1)$ meter. Consequently, the network capacity of random networks is at most $O\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec. We do not explicitly use the second arbitrary network constraint ("Interface bottleneck constraint" from Section IV-A) in the random network proof as the bounds established by that constraint are not tight, and that bound is subsumed by the bound for "destination bottleneck constraint" below.

Constraint 3 – Destination bottleneck constraint: The capacity of a multichannel network is constrained by the data that can be received by a destination node. Consider a node X which is the destination of the maximum number (that is, $D(n)$) of flows. Recall that in a (m, c)-network, each channel supports a data rate of $\frac{W}{c}$ bits/sec. Therefore, the total data rate at which X can receive data over m interfaces is $\frac{Wm}{c}$ bits/sec. Since X has $D(n)$ incoming flows, the data rate of the minimum rate flow is at most $\frac{Wm}{cD(n)}$ bits/sec. Therefore, by definition of $\lambda(n)$, $\lambda(n) \leq \frac{Wm}{cD(n)}$, implying that network capacity (which by definition is $n\lambda(n)$) is at most $O\left(\frac{Wmn}{cD(n)}\right)$ bits/sec. Substituting for $D(n)$ from Lemma 3, the network capacity is at most $O\left(\frac{Wmn \log\log n}{c \log n}\right)$ bits/sec.

The bound obtained from constraint 3 is applicable to any network, including mobile networks, as long as the destination of every flow is randomly chosen among the nodes in the network. Even when $m = c$, this bound implies that the per-flow throughput, $\lambda(n)$, is at most $O\left(\frac{W \log\log n}{\log n}\right)$ bits/sec. Previous results on capacity of mobile networks [3], [4], [26] have stated a per-flow throughput of $O(W)$ bits/sec is possible, as in their models, each node does not randomly select a destination node. Recall that in our work we choose the destination of a flow randomly from among $n - 1$ possible destinations. Considering the discussion above, the $O(W)$ bits/sec bound with mobility cannot apply when destination nodes are randomly chosen. The previous results for mobile networks hold under other models of selecting destination nodes, wherein each node is the destination of at most $O(1)$ flows (for example, such a constraint is satisfied when permutation routing is used).

Combining the above three bounds, the capacity of a random network, under channel model 1, is upper bounded by $O\left(\mathit{MIN}_O\left(W\sqrt{\frac{n}{\log n}}, W\sqrt{\frac{nm}{c}}, \frac{Wmn\log\log n}{c\log n}\right)\right)$ bits/sec. From this, we have the following theorem on the upper bound on capacity of random networks (Figure 2 has a pictorial representation).

Theorem 6: The upper bound on the capacity of a (m, c)-random network under channel model 1 is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, network capacity is $O\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.

3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $O\left(\frac{Wmn\log\log n}{c\log n}\right)$ bits/sec.

The result for channel model 2 can be similarly derived, and is given by:

Theorem 7: The upper bound on the capacity of a (m, c)-random network under channel model 2 is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, network capacity is $O\left(Wc\sqrt{\frac{n}{\log n}}\right)$ bits/sec.

2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, network capacity is $O\left(W\sqrt{nmc}\right)$ bits/sec.

3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $O\left(\frac{Wmn\log\log n}{\log n}\right)$ bits/sec.

An interesting observation from the upper bound result is that as long as $\frac{c}{m}$ is $O(\log n)$, the number of interfaces has no impact on channel capacity. This implies that when the number of channels is $O(\log n)$ (which is the common case today), there is no loss in network capacity even if each node has a single interface.

B. *Constructive lower bound*

The lower bound is established by constructing a routing scheme and a transmission schedule for any random network. The lower bound matches the upper bound implying that the bounds are tight. We will provide a construction for a $(1, c)$-network (a network wherein each node has a single interface) under channel model 1, and then invoke Lemma 2 to extend the result to a (m, c)-network. The steps involved in the construction are described next.

Cell construction

The surface of the unit torus is divided using a square grid into square cells (see Figure 5), each of area $a(n)$, similar to the approach used in [4]. The key difference in our work from [4] is that the size of the cell, $a(n)$, varies with the number of channels, and has to be carefully chosen to meet multiple constraints (which are described later in the text). In particular, we set $a(n) = \min\left(\max\left(\frac{100\log n}{n}, \frac{c}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$ as described before. Intuitively, the three values that influence $a(n)$ are based on the three constraints that were described in the upper bound proof: cell size needed to ensure connectivity, cell size needed when capacity is constrained by interference, and cell size needed when capacity is constrained by the maximum number of flows to any destination node, respectively.

We need to bound the number of nodes that are present in each cell, which is derived in Lemma 4.

Lemma 4: If $a(n) \geq \frac{100\log n}{n}$, then each cell has $\Theta\left(na(n)\right)$ nodes per cell, with high probability.

Proof: A similar result was stated in [4] without proof. Here we provide a proof based on VC-theory (see [27] for details on VC-theory), similar to the approach used by Gupta and Kumar [2]. The total number of square cells is $\frac{1}{a(n)}$. Since nodes are randomly located on the torus, the probability that any given node will lie in a specific cell is $a(n)$. We want to derive bounds on number of nodes in *every* cell in the square grid, which requires a proof of uniform convergence. The set of axis-parallel squares $\mathcal{C}$ are known to have VC-dimension 3. By applying the

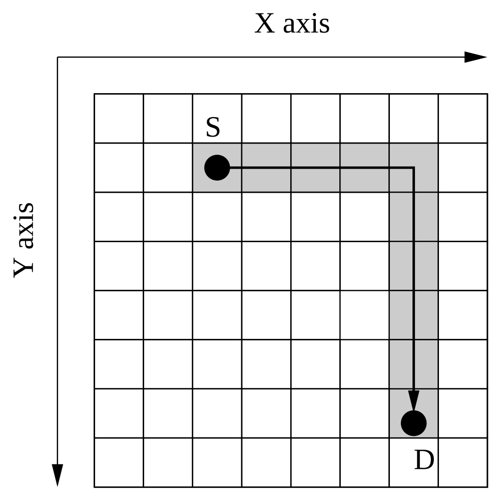

Fig. 5. Routing through cells: Packets are routed first along a row till the destination column is reached, and then along the column to the destination cell.

Vapnik-Chervonekis theorem [28], similar to the approach used in [2], we have the following bound on the number of nodes N_C in any cell C:

$$\text{Prob}\left(\sup_{C\in\mathcal{C}}\left|\frac{N_C}{n}-a(n)\right|\leq\frac{50\log n}{n}\right)>1-\frac{50\log n}{n} \tag{8}$$

where the constants in the above expression have been carefully chosen to satisfy the Vapnik-Chervonekis theorem. The above result implies that with high probability, we have

$$na(n)-50\log n\leq N_C\leq na(n)+50\log n$$

provided that $a(n)\geq\frac{100\log n}{n}$.

Hence, we can conclude that the number of nodes in any cell is $\Theta(na(n))$ with high probability, as long as $a(n)\geq\frac{100\log n}{n}$. ■

By construction, we ensure that $a(n)\geq\frac{100\log n}{n}$ for large n because $\max\left(\frac{100\log n}{n},\frac{c}{n}\right)$ is at least $\frac{100\log n}{n}$, and $\left(\frac{1}{D(n)}\right)^2$ is asymptotically at least as large as $\frac{100\log n}{n}$ as long as $D(n)=O\left(\sqrt{\frac{n}{100\log n}}\right)$. Thus, with our choice of $a(n)$, Lemma 4 holds for suitably large n, and each cell has $\Theta\left(na(n)\right)$ nodes per cell, *whp*.

The transmission range[4] of each node, $r(n)$, is set to be $\sqrt{8a(n)}$. With this transmission range, a node in one cell can communicate with any node in its eight neighboring cells. Note that when the cell size $a(n)$ increases, larger transmission range is required, as $r(n)$ is dependent on $a(n)$.

[4]Transmission range is defined to be the maximum distance over which any node can communicate.

A transmission originating from a node S interferes with another transmission from A destined to B, only if S is within a distance of $(1+\Delta)r(n)$ of receiver B (using the interference definition of protocol model). Since the distance between A and B is at most $r(n)$, the distance between the two transmitters, S and A, must be less than $(2+\Delta)r(n)$ if the transmissions were to interfere. Hence, any transmission can possibly interfere with only those transmissions from transmitters within a distance of $(2+\Delta)r(n)$. Therefore, nodes in a cell can be interfered with by only nodes in cells within a distance of $(2+\Delta)r(n)$, and this interfering area can be completely enclosed in a larger square of side $3(2+\Delta)r(n)$ (this is a loose bound). Consequently, there are at most $\frac{(3(2+\Delta)r(n))^2}{a(n)} = 72(2+\Delta)^2$ interfering cells (recall $r(n) = \sqrt{8a(n)}$). Hence, the number of interfering cells, $k_{inter} \leq 72(2+\Delta)^2$, is a constant that only depends on Δ (and is independent of $a(n)$ and n).

Routing Scheme

We use a "row-column" strategy for routing the packets (cf. [18]). A random cell is chosen as the origin of a cell co-ordinate system. Each cell is assigned X and Y co-ordinates, such that the co-ordinate values change by 1 per cell (along each axis from the origin). The X-axis is assumed to be along a line from east to west, and the Y-axis is assumed to be along a line from north to south. To route from a node in a cell with co-ordinates (x_1, y_1) to a node in a cell with co-ordinates (x_2, y_2), the packets are first sent east along the row containing the source till it intersects with the column containing the destination (i.e., follow along the row till the X-coordinate of the cell is x_2). After that, packets are sent south along the column containing the destination till the destination cell is reached. Figure 5 shows an example of the cells used to route data for a flow between source S and destination D.

In previously proposed constructions for proving lower bound on capacity [2], [4], it was immaterial which node in a chosen cell forwarded packets for some flow. However, such an approach may "*overload*" certain nodes, leading to capacity degradation, when the number of interfaces per node is smaller than the number of channels. Consequently, it is important to ensure that the routing load is distributed among the nodes in a cell. This is a key extension to the routing procedure used in earlier capacity results [2], and the extension is described next.

For each flow passing through a cell, one node in the cell is "assigned" to the flow. The assigned node of a flow in a cell is the only node in that cell which may receive/transmit data along that flow. The assignment is done using a *flow distribution procedure* as below:

Step 1 – Assign source and destination nodes: For any flow that originates in a cell, the source node S is assigned to the flow (S is necessarily in the originating cell). Similarly, for any flow that terminates in a cell, the destination node D is assigned to the flow. Since a single node in each cell is allowed to receive or transmit data for a flow, it is required that the source and destination nodes be assigned to flows originating or terminating from them.

Step 2 – Balance distribution of remaining flows: After step 1 is complete, we are left with only those flows that pass through a cell. Each such remaining flow passing through a cell is assigned to the node in the cell that has the least number of flows assigned to it so far. This step balances the assignment of flows to ensure that all nodes are assigned (nearly) the same number of flows. The node assigned to a flow will receive packets from some node in the previous cell and send the packet to a node in the next cell.

Each node is the originator of one flow. Each node is the destination of at most $D(n)$ flows, which by Lemma 3 is $\Theta\left(\frac{\log n}{\log \log n}\right)$. Therefore, step 1 of the flow distribution procedure assigns to each node at most $1 + D(n)$ flows.

We use the following lemma to bound the number of flows that pass through any cell when using the routing strategy described above.

Lemma 5: When the row-column routing is used, and $a(n) > \frac{\log n}{n}$, the maximum number of flows that pass through any cell (including flows originating and terminating in the cell) is $O\left(n\sqrt{a(n)}\right)$, with high probability.

Proof: See Appendix I for the proof. ■

The bound from the lemma always holds because by construction we ensure that $a(n) \geq \frac{100 \log n}{n}$. Step 2 of the flow distribution procedure carefully assigns the remaining flows among the nodes in the cell to ensure that all nodes end up with nearly same number of flows. By Lemma 4, each cell has $\Theta\left(na(n)\right)$ nodes, and by Lemma 5 at most $O\left(n\sqrt{a(n)}\right)$ flows pass through a cell. Therefore, step 2 will assign to any node in the network at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows. Therefore the total flows assigned to any node is at most $O\left(1 + D(n) + \frac{1}{\sqrt{a(n)}}\right)$. Based on the rules to set $a(n)$, described earlier, the maximum value of $a(n)$ is at most $\left(\frac{1}{D(n)}\right)^2$, which implies $\frac{1}{\sqrt{a(n)}}$ is at least $D(n)$. Hence, the total flows assigned to any node is always asymptotically dominated by $\frac{1}{\sqrt{a(n)}}$, and is therefore equal to $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows.

Scheduling transmissions

The transmission scheduling scheme is responsible for generating a transmission schedule for each node in the $(1, c)$-network that satisfies the following constraints:

Constraint 1: When a node X transmits a packet to a node Y over a channel j for some flow, X and Y should not be scheduled to transmit/receive at the same time for any other flow (since each node is assumed to have a single interface in the construction).

Constraint 2: Any two simultaneous transmissions on any channel should not interfere.

The multichannel construction differs from the mechanisms used in earlier constructions [2], [4] in two ways. First, the scheduling is on a per-node basis since flows are distributed among nodes, whereas in the past work it was sufficient to schedule on a per-cell basis. Second, since there is a single interface, but c channels are available (recall that we are assuming a $(1, c)$-network for now), the schedule has to additionally ensure that at most a single transmission/reception is scheduled for a node at any time (constraint 1).

We build a suitable schedule using a two-step process. In the first step, we satisfy constraint 1 by scheduling transmissions in "edge-color" slots so that at every node during any edge-color slot, at most one transmission or reception is scheduled. In the second step, we satisfy constraint 2 by dividing each edge-color slot into "mini-slots", and assigning mini-slots to channels such that any scheduled transmission is interference-free. By using the two-step process, each transmission in a mini-slot satisfies both constraint 1 and constraint 2.

Step 1 – Build a routing graph: We build a graph, called the "routing graph", whose vertices are the nodes in the network. One edge is inserted between all node pairs, say A and B, for every flow on which A and B are consecutive nodes (the routing scheme for selecting nodes along a flow was described earlier). Therefore, by this construction, every hop[5] in the network along any flow is associated with one edge in the routing graph. The resulting routing graph is a multi-graph[6] in which each node has at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ edges, since each flow through a node can result in at most two edges, one incoming and one outgoing, and we have already shown that each node is assigned to at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows. It is a well-known result [29] that a multi-graph with at most e edges per vertex can be edge-colored[7] with at most $\frac{3e}{2}$ colors. Therefore, the routing graph can be edge colored with at most some $f = O\left(\frac{1}{\sqrt{a(n)}}\right)$ colors.

We use edge coloring to ensure that when a transmission is scheduled along an edge, the interfaces on the nodes at either end of the edge are free, thereby satisfying constraint 1. We divide every 1 second period into f (which is $O\left(\frac{1}{\sqrt{a(n)}}\right)$) "edge-color" slots, each of length $\frac{1}{f}$ (which is $\Omega\left(\sqrt{a(n)}\right)$) seconds. Each of these edge-color slots is associated with an unique edge color. An edge is scheduled for transmission some time during the slot associated with its edge color (the exact duration of transmission is decided in step 2). Since edge coloring ensures that at a vertex, all edges connected to the vertex use different colors, each node will have at most one transmission/reception scheduled in any edge-color slot. By construction, each edge corresponds to a hop in the network. Therefore this scheme ensures that during every 1 second interval, along any flow in the network, one transmission is scheduled on each hop of a flow.

Step 2 – Build an interference graph: In step 2, each edge-color slot is further sub-divided into "mini-slots" as explained below, and every node has an opportunity to transmit in some mini-slot. We develop a schedule for using mini-slots, which satisfies constraint 2. The schedule decides on which mini-slot within an edge-color slot and on what channel a node may transmit, and the same schedule is used in every edge-color slot.

We build another graph, called the "interference graph", wherein, vertices are nodes in the network, and there is an edge between two nodes if they may interfere with each other. Since every cell has at most some constant

[5]A hop is a pair of consecutive nodes on a flow.

[6]A graph with possibly multiple edges between a pair of nodes.

[7]Edge-coloring requires any two edges incident on a common vertex to use different colors.

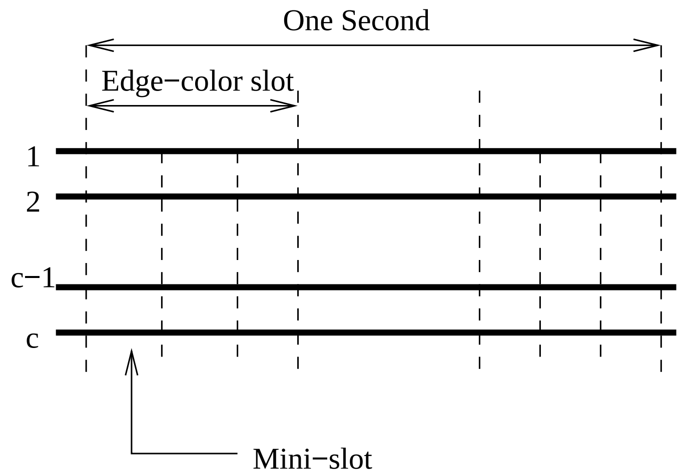

Fig. 6. Transmission schedule: Every hop along every flow is assigned to exactly one edge-color slot in each one second interval. Within the edge-color slot assigned to a hop, a specific mini-slot is chosen during which the transmitter node on that hop may transmit.

k_{inter} number of cells that may interfere with each other, and each cell has $\Theta\left(na(n)\right)$ nodes, each node has at most $g = O\left(na(n)\right)$ edges in the interference graph. It is well-known that a graph with maximum degree e can be vertex-colored[8] with at most $e+1$ colors [29]. Therefore, the graph can be vertex-colored with some $O\left(na(n)\right)$ colors, i.e., at most $k_1 na(n)$ colors for some constant k_1. Transmissions by two nodes assigned the same vertex-color do not interfere with each other. Hence, they can be scheduled to transmit on the same channel at the same time. On the other hand, nodes colored with different colors may interfere with each other, and need to be scheduled either on different channels, or at different time slots on the same channel.

We divide each edge-color slot into $\left\lceil \frac{k_1 na(n)}{c} \right\rceil$ mini-slots on every channel, and number the slots on each channel from 1 to $\left\lceil \frac{k_1 na(n)}{c} \right\rceil$. There is a total of $c\left\lceil \frac{k_1 na(n)}{c} \right\rceil$ mini-slots across the c channels. Channels are numbered from 1 to c. A node which is allocated a color $p, 1 \le p \le k_1 na(n)$ is allowed to transmit in mini-slot $\left\lceil \frac{p}{c} \right\rceil$ on channel $(p \bmod c) + 1$. The node actually transmits if the edge-coloring has allocated an outgoing edge from the node to the corresponding edge-color slot, in which case a packet is sent in that mini-slot on that outgoing edge.

Figure 6 depicts a schedule of transmissions on the network developed after the two-step scheduling process. The first step allocates one edge-color slot for each hop of every flow. The second step decides within each edge-color slot when the transmitter node on a hop may actually transmit a packet.

From step 1, each edge-color slot is of length $\Omega\left(\sqrt{a(n)}\right)$ seconds. From step 2, each edge-color slot is sub-divided into $\left\lceil \frac{k_1 na(n)}{c} \right\rceil$ mini-slots. Therefore, each mini-slot is of length $\Omega\left(\frac{\sqrt{a(n)}}{\left\lceil \frac{k_1 na(n)}{c} \right\rceil}\right)$ seconds. Each channel can transmit at the rate of $\frac{W}{c}$ bits/second. Hence, in each mini-slot, $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{c\left\lceil \frac{k_1 na(n)}{c} \right\rceil}\right)$ bits can be transported.

[8]Vertex-coloring requires any two vertices sharing a common edge to use different colors.

Since $\left\lceil \frac{k_1 na(n)}{c} \right\rceil \leq \frac{k_1 na(n)}{c} + 1$, we have, $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{k_1 na(n)+c}\right)$ bits/sec. Depending on the asymptotic order of c, either $na(n)$ or c will dominate the denominator of $\lambda(n)$. Hence, $\lambda(n) = \Omega\left(MIN_O\left(\frac{W}{n\sqrt{a(n)}}, \frac{W\sqrt{a(n)}}{c}\right)\right)$ bits/sec. Since each flow is scheduled to receive one mini-slot on each hop during every 1 second interval, every source-destination flow can support a per-node throughput of $\lambda(n)$ bits/sec. Therefore, the total network capacity is equal to $n\lambda(n)$ which is equal to $\Omega\left(MIN_O\left(\frac{W}{\sqrt{a(n)}}, \frac{Wn\sqrt{a(n)}}{c}\right)\right)$ bits/sec.

Recall that $a(n)$ is set to $\min\left(\max\left(\frac{100\log n}{n}, \frac{c}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$. Substituting for $a(n)$ (the three possible values of $a(n)$ gives rise to three capacity regions) in the equation for capacity (derived above), we have the result:

Theorem 8: The achievable capacity of a $(1, c)$-random network under channel model 1 is as follows:

1) When c is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When c is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\frac{c}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{c}}\right)$ bits/sec.
3) When c is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\left(\frac{\log\log n}{\log n}\right)^2\right)$, and the network capacity is $\Omega\left(\frac{Wn\log\log n}{c\log n}\right)$ bits/sec.

Using Lemma 2, the results for a (m, c)-network can be obtained by replacing every usage of c in Theorem 8 by $\frac{c}{m}$. Therefore, we have:

Theorem 9: The achievable capacity of a (m, c)-random network under channel model 1 is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\frac{c}{mn}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.
3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\left(\frac{\log\log n}{\log n}\right)^2\right)$, and the network capacity is $\Omega\left(\frac{Wmn\log\log n}{c\log n}\right)$ bits/sec.

The result for channel model 2 can be similarly derived, and is given by:

Theorem 10: The achievable capacity of a (m, c)-random network under channel model 2 is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(Wc\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\frac{c}{mn}\right)$, and the network capacity is $\Omega\left(W\sqrt{nmc}\right)$ bits/sec.
3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\left(\frac{\log\log n}{\log n}\right)^2\right)$, and the network capacity is $\Omega\left(\frac{Wmn\log\log n}{\log n}\right)$ bits/sec.

The lower bound matches the upper bound implying that the bounds are tight. Recall that the transmission range $r(n)$ has been set to $\sqrt{8a(n)}$. Hence, the *transmission range is larger* in case 2 and case 3 of Theorem 9 as compared to case 1 (since $a(n)$ increases). This implies that in multichannel networks with large number of channels, higher transmission power is necessary for meeting capacity bounds than is required in a single channel network.

C. Capacity results with other traffic models

The multichannel network capacity under the random destination model was stated in Theorem 10. The constructions and the results are applicable to some other traffic models that have been proposed in the literature. Alternate traffic models may result in different values of $D(n)$ (recall that $D(n)$ is the maximum number of flows for which a given node is the destination). The capacity results presented before can be restated in terms of $D(n)$. However, the results hold for those traffic models where it is equally likely for the destination node of any flow to be in any cell (this is required for row-column routing to work). In addition, the results hold only if $D(n)$ is not too large. Specifically, recall that $a(n)$ has been chosen such that it is at most $\left(\frac{1}{D(n)}\right)^2$, and $a(n)$ should also be at least $\frac{100 \log n}{n}$. This implies that a valid $a(n)$ can be chosen only if $D(n) \leq \sqrt{\frac{n}{100 \log n}}$. Under these restrictions on traffic models, the capacity of a multichannel network is:

Theorem 11: The capacity of a (m, c)-random network under channel model 1 as a function of $D(n)$ is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{1}{D(n)}\right)^2\right)$, $a(n) = \Theta\left(\frac{c}{mn}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.
3) When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{1}{D(n)}\right)^2\right)$, $a(n) = \Theta\left(\left(\frac{1}{D(n)}\right)^2\right)$, and the network capacity is $\Omega\left(\frac{Wmn}{cD(n)}\right)$ bits/sec.

Gupta and Kumar [2] choose the destination for each source node by first picking a random point, and then selecting the node closest to this point as the destination. For this model of selecting destinations, it has been shown [30] that $D(n) = \Theta(\log n)$. This traffic model meets the requirements of Theorem 11 (as destinations are randomly located, and $D(n)$ is small enough). Therefore, the capacity results for Gupta and Kumar model can be obtained by substituting $D(n) = \Theta(\log n)$ in Theorem 11. The lower bound for multichannel networks under the Gupta and Kumar traffic model can also be proved using an alternate routing approach, called "straight-line routing" (which we used in [1]), instead of using the row-column routing. However, straight-line routing may not hold under the random traffic model used in this paper (where each node chooses a random node as the destination, instead of picking a node closest to a random point). The straight-line routing proof uses some results from [4] which have not been proved for the version of random traffic model used in this report.

In another traffic model considered in literature, $\frac{n}{2}$ nodes are designated as sources, and the remaining $\frac{n}{2}$ nodes are designated as destinations. A random one-to-one mapping is set up between source and destination nodes. In

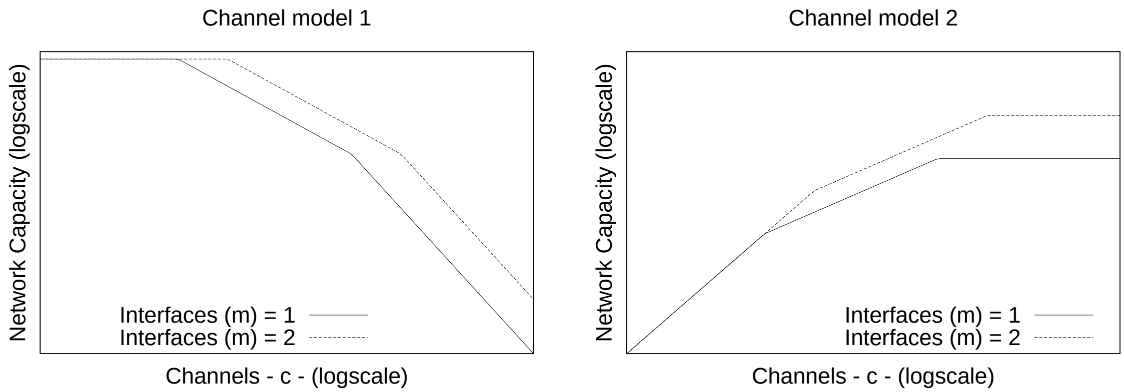

Fig. 7. Example plot of network capacity as the number of channels is scaled. Capacity values are normalized to capacity in a $(1, 1)$-network.

this model, $D(n) = 1$, and destinations are randomly located because nodes are still placed uniformly at random on the torus, and the mapping between sources and destinations is random. Therefore, the capacity results under this traffic model can be derived by replacing $D(n) = 1$ in Theorem 11..

Permutation routing is yet another traffic model considered in literature. Under permutation routing, each node is the source of exactly one flow and also the destination of exactly one flow. We discuss this traffic model further in Section VII. Under permutation routing, $D(n) = 1$ and destinations are randomly located. Therefore, results under permutation routing model can also be derived using Theorem 11.

D. Implications

Figure 7 plots the network capacity as the number of channels in the network is scaled, for a one interface and a two interface network. The figure plots the scaling with some fixed n, without accounting for constants, for the two channel models. As we can see from the figure, under the channel model 1, the total bandwidth is fixed, and the network capacity reduces when the number of channels increases. In contrast, under channel model 2, bandwidth is added when the number of channels increases, thereby increasing network capacity (up to a point). Note that the results under the two models are not contradictory, because the capacity always degrades with more channels when compared to the capacity in a (c, c)-network. Furthermore, when the number of interfaces is increased, there is no improvement in capacity as long as $\frac{c}{m}$ is $O(\log n)$, but beyond that threshold, adding more interfaces improves capacity. Since the curves in Figure 7 plot the number of channels on the X-axis, the threshold $\frac{c}{m}$ is achieved for a larger value of c when m is increased.

Figure 8 plots the network capacity as the ratio of channels to interfaces, $\frac{c}{m}$, is increased for a one interface and a two interface network. When the number of interfaces is increased by some factor of k, then for the any given ratio of $\frac{c}{m}$, this implies that the number of channels is also increased by a factor of k. Since the total bandwidth is fixed under channel model 1, a k-fold increase in channels does not increase network capacity (and therefore,

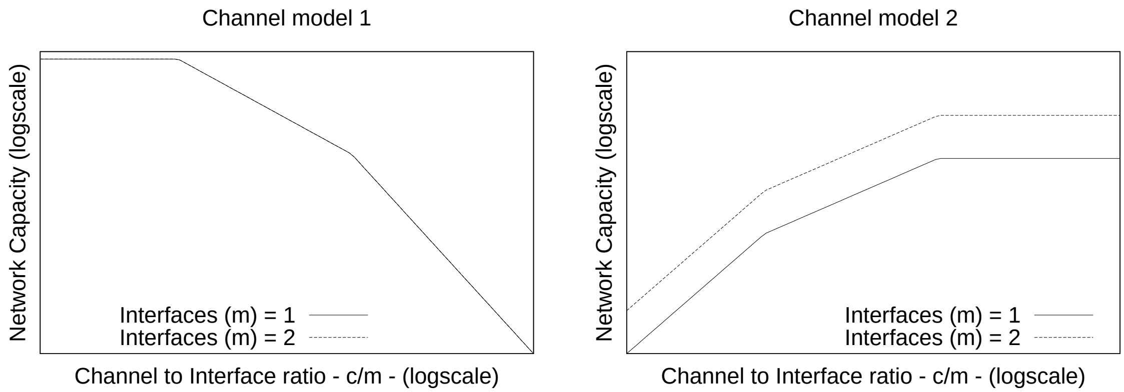

Fig. 8. Example plot of network capacity as the ratio between channels to interfaces is scaled.Capacity values are normalized to capacity in a $(1, 1)$-network. The two curves under channel model 1 overlap.

the curves for different number of interfaces overlap in Figure 8). However, since the bandwidth per channel is fixed under channel model 2, a k-fold increase in channels implies a k-fold increase in bandwidth, leading to a k-fold increase in the network capacity. In addition, under both channel models, the boundaries of the three capacity regions depend on $\frac{c}{m}$.

The results imply that the capacity of multichannel random networks with total channel data rate of W is the same as that of a single channel network with data rate W as long as the ratio $\frac{c}{m}$ is $O(\log n)$. When the number of nodes n in the network increases, we can also scale the number of channels (for example, by using additional bandwidth, or by dividing available bandwidth into multiple sub-channels). Even then, as long as the channels are scaled at a rate not more than $\log n$, there is no loss in capacity even if a single interface is available at each node. In particular, if the number of channels c is a fixed constant, independent of the node density, then as the node density increases beyond some threshold density (at which point $c = O(\log n)$), there is no loss in capacity even if just a single interface is available per node. Thus, this result may be used to roughly estimate the number of interfaces each node has to be equipped with for a given node density and a given number of channels.

In a single channel random network, i.e., a $(1, 1)$-network, the capacity bottleneck arises out of the channel becoming fully utilized, and not because interface at any node is fully utilized. On an average, the interface of a node in a single channel network is busy only for $\frac{1}{X}$ fraction of the time, where X is the average number of nodes that interfere with a given node. In a $(1, 1)$-random network with n nodes, each node on an average has $\Theta(\log n)$ neighbors to maintain connectivity [2]. This implies that in a single channel network, each interface is busy for only $\Theta\left(\frac{1}{\log n}\right)$ time. Our construction utilizes this slack time of interfaces to support up to $O(\log n)$ channels without loss in capacity. In general, the loss in capacity in a random network is a function of the number of channels and the number of nodes in a neighborhood[9].

In earlier capacity results [2], [4], the transmission range, and therefore the neighborhood size, is a function of only the node density. However, for multichannel networks, the transmission range has to be chosen based on ratio of channels to interfaces, in addition to the node density. For example, with a given node density, when the ratio of number of channels to number of interfaces is large (specifically, $\omega(\log n)$), the number of interfaces in a neighborhood will be smaller than the total number of channels. Therefore, even if all the interfaces are being used continuously, it is not possible to fully saturate the available channels. This can result in significant capacity degradation.

The capacity degradation can be reduced by increasing the size of a neighborhood, thereby ensuring that the number of interfaces in a neighborhood is equal to the number of channels. Therefore, the lower bound construction requires the cell size to be chosen such that the number of interfaces (or nodes, when each node has a single interface) in each neighborhood is greater than or equal to the number of channels. Hence, it turns out that the optimal strategy for maximizing capacity when number of channels is large is to sufficiently increase the cell size $a(n)$, which implies that a *larger transmission range* $r(n)$ *is needed* to allow communication with neighboring cells. However, there is still some capacity loss because larger transmission range (than that is needed for connectivity alone) lowers capacity by "consuming" more area. In summary, in a single channel random network, the transmission range is chosen to be large enough to ensure connectivity. However, in the case of multichannel networks, the transmission range has to be chosen such that it is sufficiently large to ensure that all channels are utilized, in addition to guaranteeing connectivity.

VI. Impact of switching delay

The previous discussion on multichannel capacity has not considered the impact of interface switching delay. When the number of interfaces at each node is smaller than the number of channels, interfaces may have to be switched between channels. Switching an interface from one channel to another may incur a switching delay, say S. For example, existing IEEE 802.11-based wireless interfaces require [23] between few tens to hundreds of microseconds to switch from one channel to another. However, switching delay is independent of the number of nodes in the network.

We will show that if there are no end-to-end delay constraints, switching delay will not affect network capacity. For this, we use the end-to-end delay constraint definition from [4]. Each packet is assumed to have a size L, and L is scaled with respect to the throughput obtained for each end-to-end flow. If each flow can transport λ bits/sec, then each flow is assumed to send packets of size $L = \lambda$. In the lower bound construction provided before, if packet sizes are set to λ bits, each packet traverses at least one hop in one second. Therefore, the end-to-end delay

[9] The neighborhood of a node consists of all other nodes that may interfere with it.

of a flow will be bounded by the number of hops on the flow, when there is no interface switching latency. Let us assume that the minimum end-to-end delay in the absence of interface switching latency is D_{opt}. A reasonable delay constraint in the presence of switching latency is to require that the end-to-end delay is at most a small constant multiple of D_{opt}; otherwise applications may see a large increase in the end-to-end delay. This requirement may be equivalently translated to allow a maximum packet size of L.

A. Capacity in the absence of end-to-end delay constraints

In the case of arbitrary networks, capacity bounds are met without requiring interface switching at all (as was shown in the construction used for lower bound). Hence, switching delay will not impact the capacity of arbitrary networks, even if there is an end-to-end delay constraint. In the absence of any end-to-end delay constraints, we show next that the capacity of random networks is independent of switching delay (the construction is described next).

In the construction we use to establish lower bound for random networks, interfaces may have to be switched between channels (when receiving data). In the worst case, an interface may have to be switched between channels for every packet transmission. If there is no end-to-end delay constraint, then we propose a simple "guard slot" approach which ensures that capacity loss can be made arbitrarily small even in the presence of switching delay.

The "guard slot" approach is as follows. Suppose that each packet is L bits long. This implies that the length of each edge color slot is $T = \frac{Lc}{W}$ seconds (since each channel supports a data rate of $\frac{W}{c}$ bits/sec under channel model 1). One simple way of hiding the interface switching delay S is to insert a "guard" slot of duration S between two "edge-color" slots during which all channels are idle, to ensure that there is sufficient time for interface switching. With this approach, the network capacity will be only $\frac{T}{T+S}$ fraction of the capacity when there is no switching delay. However, the capacity reduction can be made arbitrarily small by sending extremely large packets ($L \gg \lambda$) resulting in $T \gg S$, leading to large end-to-end delay. Therefore, in the absence of end-to-end delay constraints, by using large data packets, the capacity degradation in random networks can be made arbitrarily small.

B. Capacity in the presence of end-to-end delay constraints

From prior discussions, even in the presence of delay constraints, the capacity of arbitrary networks is not affected by switching delay, since switching is not required to meet the capacity bounds. In the case of random networks as well, the upper bound proofs do not mandate interfaces to be switched, and therefore, even with switching delay, there may be no change in the capacity. However, so far we have not addressed the question whether the capacity of random networks is independent of the switching delay when there are end-to-end delay constraints.

In the presence of end-to-end delay constraints, switching delay *does reduce* the achievable network capacity in the lower bound constructions proposed earlier. For example, considering the guard-slot approach described above, when there is a restriction on the maximum packet size, each edge-color slot is bounded by some length T, and the network capacity will be only $\frac{T}{T+S}$ of the capacity without switching delay, We next describe an approach that shows using additional interfaces at each node is *sufficient* in many scenarios to hide the switching delay, even with end-to-end delay constraints.

The new approach simulates a *virtual interface* having zero switching delay using multiple physical interfaces that each have a switching delay S. By this construction, the use of $v-1$ additional interfaces per node can hide the switching delay, i.e., a (v, c)-network using interfaces with switching delay S can achieve the same capacity and end-to-end delay bounds as a $(1, c)$-network using one interface with 0 switching delay. This construction suggests that multiple interfaces are *sufficient* to overcome the impact of switching delay, though multiple interfaces may not be *necessary*.

Lemma 6: Suppose that the time required for packet transmission in a $(1, c)$-network is $T = \frac{Lc}{W}$, and suppose $v = \left\lceil \frac{S}{T} \right\rceil + 1$. Then a (v, c)-network built with interfaces having switching delay S, can achieve the same capacity and end-to-end delay as a $(1, c)$-network built with interfaces having 0 switching delay.

Proof: Let us assume that each node has $v = \left\lceil \frac{S}{T} \right\rceil + 1$ interfaces, each having a switching delay S. We build a *virtual interface* with zero switching delay by using the v physical interfaces, as shown in Figure 9. We consider any time interval of length vT. We divide this time into v slots of length T, and only allow the i^{th} interface, $1 \leq i \leq v$, to transmit/receive in slot i. Thus, each physical interface is used for transmission/reception in one slot, and is idle for the next $(v-1)$ slots of total duration $(v-1)T$ seconds. Since $v = \left\lceil \frac{S}{T} \right\rceil + 1$, we have:

$$\begin{aligned} (v-1)T &= \left\lceil \frac{S}{T} \right\rceil T \\ &\geq S \end{aligned}$$

Hence, between two successive operations of a physical interface there is at least a gap of S, which ensures that switching delay is provisioned for. By this construction, the simulated virtual interface can continuously transmit/receive, with 0 switching delay. Therefore, a network using v interfaces having switching delay S, can mimic the behavior of a $(1, c)$-network built with interfaces having switching delay 0. ■

From the previous lemma, by increasing the number of interfaces at each node by a factor of v, switching delay is completely hidden. We next discuss the capacity implications of using v physical interfaces at each node to construct a virtual interface, instead of directly using the v interfaces to send data in parallel.

From Theorem 9, we note that when the number of channels is $O(\log n)$ and there is no switching delay, the capacity of a (v, c)-network is the same as that of a $(1, c)$-network. Using this observation along with Lemma 6, we can conclude that by using the virtual interface technique, the capacity of a (v, c)-network with each interface having switching delay S is the same as the capacity of a (v, c)-network with each interface having switching delay

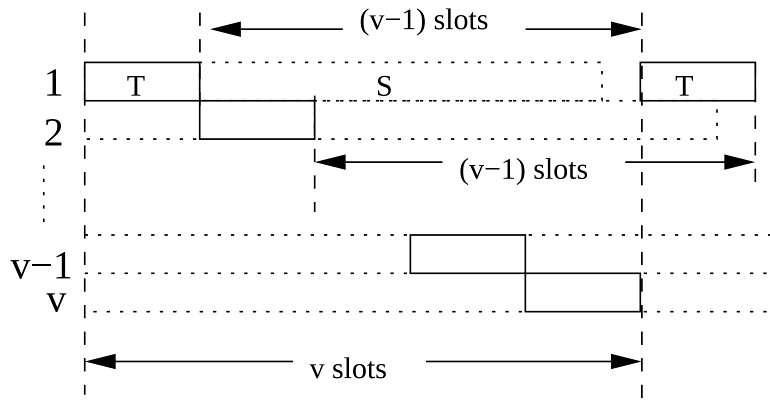

Fig. 9. Constructing one virtual interface with zero switching delay by using v physical interfaces with switching delay S. Each packet transmission requires T seconds.

0. Hence, when the number of channels is $O(\log n)$, which is a scenario of significant practical interest, *there is no capacity loss even with switching delay, provided multiple interfaces are used.*

Again, from Theorem 9, we note that when the number of channels is larger ($\Omega(\log n)$) and there is no switching delay, the capacity of a $(1, c)$-network is *lower* than that of a (v, c)-network. Hence, using this observation along with Lemma 6, we can conclude that using the virtual interface technique when the number of channels is larger ($\Omega(\log n)$), a (v, c)-network with each interface having switching delay S *will have lower capacity* than a (v, c)-network with each interface having switching delay 0. Using Theorem 9, we can show that for this scenario, the capacity will be lower by a factor of $\frac{1}{\sqrt{v}} \approx \sqrt{\frac{T}{T+S}}$ (since $v \approx \frac{T+S}{T}$) when number of channels is between $\Omega(\log n)$ and $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, and by a factor of $\frac{1}{v} \approx \frac{T}{T+S}$ when number of channels is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$. In contrast, if the guard slot approach is used, the capacity is lower by a factor $\frac{T}{T+S}$ in all cases, independent of the number of channels. Therefore, although there is a capacity loss with switching delay for certain scenarios using the virtual interface technique, it is still significantly better than the guard slot approach when the number of channels is small.

C. Other constructions

The constructions used to establish lower bound on capacity potentially require interface switches at several hops of a flow. Alternate constructions are possible [31] such that an interface switch is required on at most one hop of each flow. Such a construction may reduce the number of switches required in the network, and could be used to reduce the impact of switching delay.

The capacity analysis has assumed that each node has one flow active all the time. Typically, in deployed networks, every node may not have an active flow all the time. In such a scenario, nodes without active flows could still forward data for other nodes, and the interfaces of the inactive nodes may be viewed as “spare” interfaces that are available in the network. Such spare interfaces could also be used to hide interface switching delay, instead of requiring additional interfaces at each node to hide the switching delay. We defer a formal study on hiding interface switching delay by using spare interfaces to future work.

VII. Capacity with fixed interfaces

In the previous section, we considered the capacity of multichannel networks when switching interfaces incurs a delay. In this section, we study the capacity of multichannel networks when interfaces do not switch at all. We assume that each interface is fixed to some channel, and the channel to which an interface is fixed can be set by the network designer. It may be beneficial to keep interfaces fixed when the interface switching delay is large. For tractability, we study the capacity problem under a slightly different model for selecting source-destination pairs, called the permutation traffic model. Under this model, we show that there is a degradation in capacity, proportional to the number of channels, when there is a single interface per node, and interfaces are not allowed to switch. However, the capacity degradation can be prevented if each node is equipped with two interfaces (and interfaces continue to be fixed on some channels).

A permutation is an one-to-one correspondence from a set $\{1, 2, .. n\}$ to itself. There are $n!$ possible permutations, and a random permutation is defined as a permutation chosen uniformly at random from all the possible permutations. The permutation routing model assumes that the source-destination pairs are chosen as a random permutation. This implies that each node is the source of exactly one flow, and each node is the destination of exactly one flow. In contrast, the traffic model used earlier in this report allowed nodes to be destinations of more than one flow. Permutation routing model has been assumed by other works on capacity in the past (e.g. [7], [8]), and is simpler to analyze.

Under the permutation traffic model, the maximum number of flows per destination, $D(n) = 1$. Recall from the discussions in Section V-C, that the capacity results for permutation model can be obtained by substituting $D(n) = 1$ in Theorem 11. Therefore, the random network capacity under permutation routing model is given by,

Theorem 12: The capacity of a (m, c)-random network under channel model 1, and permutation traffic model, is as follows:

1) When $\frac{c}{m}$ is $O(\log n)$, the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O(n)$, the network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.
3) When $\frac{c}{m}$ is $\Omega(n)$, the network capacity is $\Omega\left(\frac{Wnm}{c}\right)$ bits/sec.

A. Capacity bound with a single fixed interface

When every node has a single fixed interface, nodes fixed on a certain channel cannot communicate with nodes fixed on any other channel. Network capacity depends on the smallest throughput obtained by any flow, and to ensure that the network capacity is greater than zero, any pair of source-destination nodes must be fixed to a common channel. This constraint precludes fixing nodes to channels arbitrarily.

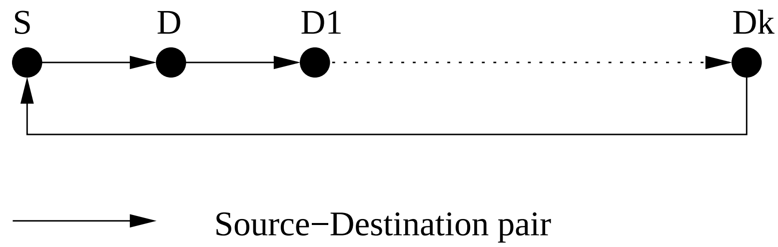

Fig. 10. Example of a cycle formed by source-destination pairs under the permutation traffic model.

Let us suppose that each node has been fixed on a channel, and this channel assignment has been done while ensuring that any source-destination pair uses the same channel. Let a channel $i, 1 \leq i \leq c$, have n_i nodes fixed on it, and let λ_i be the smallest flow throughput among the flows on channel i. Consider some channel i. Now, the n_i nodes must still satisfy the connectivity constraint to ensure that no nodes are disconnected from each other. As before, we assume that all nodes use a common transmission range. Therefore, the transmission range on channel i must be at least as large as the transmission range required when all nodes share a common channel, i.e., transmission range $r(n_i) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$. Furthermore, the average distance between the source-destination pairs on channel i continues to be $\Theta(1)$ meters. Using upper bound results from Gupta and Kumar [2], per-flow throughput of a network having n_i nodes using transmission range $r(n_i)$, and channel bandwidth $\frac{W}{c}$ (we assume channel model i) is upper-bounded as follows:

$$\lambda_i = O\left(\frac{W}{c}\frac{1}{n_i r(n_i)}\right) \tag{9}$$

Substituting for $r(n_i)$, we get

$$\lambda_i = O\left(\frac{W}{cn_i}\sqrt{\frac{n}{\log n}}\right)$$

By definition, the network-wide per-flow throughput, λ, is defined to be the minimum throughput achieved by any flow, i.e., $\lambda = \min_i(\lambda_i)$. Therefore, λ is limited by the per-flow capacity in the channel having the maximum number of nodes. Using this observation, the network capacity $n\lambda$ is given by,

$$n\lambda = O\left(\frac{Wn}{c\max_i(n_i)}\sqrt{\frac{n}{\log n}}\right) \tag{10}$$

We now estimate the value of $\max_i(n_i)$. Recall our requirement that if a source node S is assigned to a channel, then its corresponding destination D should be assigned to the same channel. In turn, the destination of node D, say D1, should also be assigned the same channel. This process continues till the destination of one of the nodes is the first node S. Therefore, the source-destination assignments form a cycle as shown in Figure 10. Hence, the value of $\max_i(n_i)$ is equal to the size of the largest cycle in a random permutation.

It is well-known [32] that the probability a random permutation has a cycle of length $m, m > \frac{n}{2}$, is $\frac{1}{m}$. Therefore, the probability that a permutation has a cycle of length greater than $\frac{n}{2}$ is given by [33],

$$\begin{aligned} \text{Prob(cycle length greater than } \tfrac{n}{2}) &= \sum_{m=\frac{n}{2}+1}^{n} \frac{1}{m} \\ &\simeq \ln(n) - \ln\left(\frac{n}{2}\right) \quad \text{(for large n)} \\ &= \ln(2) \\ &\simeq 0.69 \end{aligned}$$

Therefore, $\max_i(n_i)$ is at least $\frac{n}{2}$ with a non-zero probability exceeding a constant (independent of n). This implies that an upper bound on network capacity can be obtained by replacing $\max_i(n_i)$ by $\frac{n}{2}$ in Equation 10 giving,

$$n\lambda = O\left(\frac{W}{c}\sqrt{\frac{n}{\log n}}\right) \tag{11}$$

This bound can be shown to be tight. A simple construction is to assign all nodes to a common channel, independent of the total channels available. Then, the common channel can support a data rate of $\frac{W}{c}$, and the network can be operated using the Gupta and Kumar construction for a single channel network. This construction yields the same capacity as specified by the upper bound in Equation 11, proving the upper bound is tight.

Comparing Equation 11 with Theorem 12, we can see that keeping the single interface at a node fixed results in a capacity loss. The loss can be as large as $\frac{1}{c}$ in the first capacity region (when $c = O(\log n)$). Therefore, this clearly suggests that if each node has a single interface, switching interfaces is necessary to avoid capacity degradation.

The random traffic model used earlier in the paper requires each node to randomly select a destination. Consider a graph built from the random traffic model, where vertices are nodes in the network, and two vertices are connected by an undirected edge if their corresponding nodes form a source-destination pair. In this graph, each vertex has an average degree of 2 (the graph has n edges, resulting in an average vertex degree of 2). A similar graph, called the random graph [34], can be constructed by taking n vertices and choosing every edge between vertices with a probability of $\frac{2}{n}$. The resultant graph also has an average vertex degree of 2, but is not identical to the graphs formed by the random traffic model. It has been shown that random graphs have a connected component of size $\Theta(n)$ when their average degree is greater than 1. Therefore, if source-destination pairs where chosen using the random graph approach, even then the network capacity would follow Equation 11. We speculate that the bound of Equation 11 also applies to the random traffic model considered earlier in the paper, though we do not have a proof to support the conjecture.

B. Lower bound with two fixed interfaces: permutation routing model

In the previous section, we showed that if nodes have a single interface, and if channel assigned to the interface is fixed, then there is a capacity degradation. In this section, we show that if all nodes have two interfaces, and even if channels assigned to the interfaces are fixed, there is no loss in capacity, under the permutation traffic model. We designate the first interface at each node as the "primary interface", and the channel assigned to the first interface as the "primary channel". Similarly, the second interface is designated as the "secondary interface". The key idea here is that different nodes in a cell are assigned the primary channels such that in a cell, all channels have the same number of primary interfaces fixed to them. The secondary interface of a node is fixed to the primary channel of the node's destination. Data is sent from a source to a destination on the primary channel of the destination (over all hops). In the rest of this section, we show that this construction is feasible and achieves the same capacity as a network with one interface that can switch.

As before, the surface of the unit torus is divided into square cells each of area $a(n)$. We assume that c is at most n. Since the destination bottleneck constraint is not present in the random permutation model, we choose the area $a(n)$ slightly differently from before. Specifically, we set $a(n) = \max(\frac{100 \log n}{n}, \frac{c}{n})$. From Lemma 4, we know that each cell has $\Theta(na(n))$ nodes with high probability. Therefore, every cell has at least $k_2 na(n)$ nodes, where k_2 is a constant. If $k_2 na(n) < c$, we increase the area $a(n)$ by a factor of $\frac{c}{k_2 na(n)}$, and this factor can always be bounded by a constant (because $c = \Theta(na(n))$). The scaling ensures that every cell has at least c nodes. From now on, we assume that we are considering the scaled cells. Within each (scaled) cell there are at most $k_3 na(n)$ nodes, where k_3 is a constant. We number these nodes from 1 to $k_3 na(n)$, and assign node i to channel $(i \bmod c) + 1$. With this assignment there are $n_c = \Theta\left(\frac{na(n)}{c}\right)$ nodes per channel in each cell.

We continue to use the row-column routing technique that was used earlier in the paper. Figure 11 shows the routing scheme. Recall that in row-column routing, packets are first sent east along the row containing the source till it intersects with the column containing the destination. After that, packets are sent south along the column containing the destination till the destination cell is reached.

Let us consider the traffic going through some cell L on some channel i. There are $\frac{1}{\sqrt{a(n)}}$ cells per column, and therefore, each column will have $\frac{n_c}{\sqrt{a(n)}}$ nodes on channel i. Since each node is the destination of exactly one flow under the permutation routing model, cell L will forward at most $\frac{n_c}{\sqrt{a(n)}}$ flows that are headed along the column containing L toward their destinations.

The network has $\frac{n_c}{a(n)}$ total destination nodes (there are $\frac{1}{a(n)}$ cells, and at most n_c nodes on a channel in each cell) receiving data on channel i. We now want to bound the number of source nodes sending data on channel i on any row. Since node locations are chosen independently at random, it is equally probable that the source node corresponding to a destination node is in any given row. Therefore, the number of source nodes in any given row

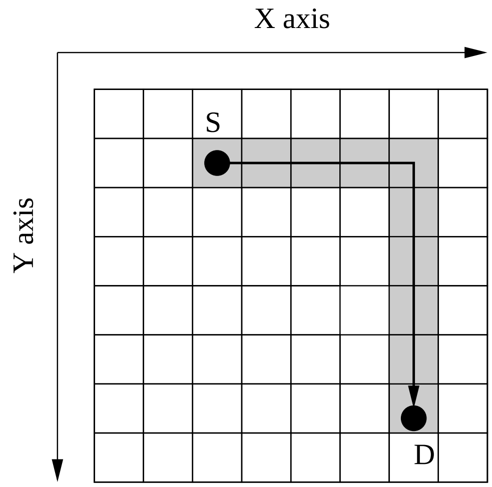

Fig. 11. Routing through cells: Packets are routed first along a row till the destination column is reached, and then along the column to the destination cell.

may be modeled as a "Balls and Bins" problem where $\frac{n_c}{a(n)}$ balls are thrown into $\frac{1}{\sqrt{a(n)}}$ bins. Results from [25] show that if x balls are thrown into y bins, and $x \geq y \log y$, then each bin has $\Theta\left(\frac{x}{y}\right)$ balls w.h.p. Since in our case $x = \frac{n_c}{a(n)}, y = \frac{1}{\sqrt{a(n)}}$, and $n_c \geq 1$, implying $x \geq y^2$, each row will have $\Theta\left(\frac{n_c}{\sqrt{a(n)}}\right)$ source nodes. Therefore, as each source node originates one flow, the number of flows on any channel that pass through a cell L while traversing the row containing L (i.e., flows originate at some source node along the row containing L), is bounded by $\Theta\left(\frac{n_c}{\sqrt{a(n)}}\right)$.

Adding the flows along rows and columns, the total flows on any channel i passing through any cell L is $\Theta\left(\frac{n_c}{\sqrt{a(n)}}\right)$. Since each cell has n_c nodes on channel i, if the flows are carefully balanced across nodes, the number of flows per node is given by $\Theta\left(\frac{1}{\sqrt{a(n)}}\right)$. Using the Gupta and Kumar [2] construction for a single channel, each node receives a fraction $\Theta\left(\frac{1}{n_c}\right)$ of the channel time (as there are $\Theta(n_c)$ nodes on any channel in a neighborhood). Therefore, each flow receives a throughput of $\Theta\left(\frac{W}{c}\frac{\sqrt{a(n)}}{n_c}\right)$ bits/sec. Substituting for n_c, and multiplying by n, the network capacity is $\Theta\left(W\frac{1}{\sqrt{a(n)}}\right)$ bits/sec. Substituting for $a(n)$, we have the following theorem:

Theorem 13: The capacity of a random network with two fixed interfaces per node under channel model 1, and permutation traffic model, is as follows:

1) When c is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.
2) When c is $\Omega(\log n)$ and also $O(n)$, $a(n) = \Theta\left(\frac{c}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{c}}\right)$ bits/sec.

The construction presented above could be easily generalized when more than two interfaces are available (by grouping interfaces using Lemma 2). Similarly, the constructions can be extended to the scenario with $c = \Omega(n)$ channels (by applying the earlier construction, but using only n channels), for which the network capacity is given by $\Omega\left(\frac{Wn}{c}\right)$ bits/sec. Comparing with Theorem 12, we see that there is no loss in capacity even if interfaces are fixed, provided the number of interfaces is doubled.

C. *Lower bound with two fixed interfaces: random routing model*

The lower bound construction used for the permutation routing model is also applicable under the random routing model. Therefore, it can be shown that there is no loss in network capacity for up to $O(\log n)$ channels even under the random traffic model.

The results for random traffic model can also be proved by using the construction techniques from [31]. In this approach, as before, the primary interfaces in each cell are carefully assigned to channels to balance the interfaces across all channels. The secondary interface at each node is independently assigned to a random channel, which is the key difference from the earlier construction. Traffic is initially sent from the source to the destination on the secondary channel, say s (at each hop, a node with primary interface tuned to channel s relays the packet). When the packet is within $c\log n$ hops of the destination[10], then the packet enters a transition phase. During the transition phase, if the packet is at a node that has its primary channel on s and the secondary channel on the destination's primary channel, say d, then the packet is sent out on channel d. After the packet transitions to the destination's primary channel, it is relayed on that channel till it reaches the destination. This construction can be shown to achieve the same network capacity as a network where interfaces can switch (see constructions in [31] for more details). Hence, even under the random traffic model, two fixed interfaces per node is sufficient to achieve asymptotically optimal capacity.

VIII. DISCUSSIONS

The theoretical analysis has yielded the capacity of wireless networks with the number of channels varying across a wide range. The region where the number of channels is scaled as $O(\log n)$ seems to be of immediate practical interest, since the number of channels provisioned for in current wireless technologies is not too large. However, there are many recent efforts aimed at utilizing frequency spectrum in higher frequency bands, where significantly larger bandwidth is available for use. For example, there is around 7 GHz of spectrum available for unlicensed use in the 60 GHz band [35], whereas the total bandwidth used in current wireless technologies, such as IEEE 802.11, is less than 500 MHz. The bandwidth that may become available in higher frequency bands can be split up into a large number of channels, and therefore the region with number of channels greater than $\Omega(\log n)$ may be of practical interest in the near future.

The capacity analysis has shown that a single interface may suffice for random networks with up to $O(\log n)$ channels. The capacity-optimal lower bound construction used to support the above claim is based on certain

[10]If a route requires fewer than $c\log n$ hops, then the route length is intentionally increased by using a detour [31].

assumptions, all of which may not be satisfied in practice. For example, we assume that interface switching delay is zero, transmission range of interfaces can be carefully controlled, and there is a centralized mechanism for co-ordinating route assignment and scheduling. In addition, the theoretical analysis derives asymptotic results, and capacity can be improved by constant factors in the lower bound constructions by using multiple interfaces. From Section VI, we note that when interface switching delay is not zero, having more than one interface may be beneficial. Furthermore, protocol design has identified many benefits of using at least two interfaces at each node, such as allowing full-duplex transfer, and simplifying the development of distributed protocols for utilizing multiple channels, as seen in our other work.

Our simulation and testbed experiments [36] have shown that having more than one interface may be beneficial in practice. However, these experiments do not prove multiple interfaces are necessary for obtaining all the observed performance improvement. In addition, our simulation results also show that it is not necessary to have one interface per channel to utilize all the channels, and in fact even many (e.g., 12) channels can be fully utilized by using only two interfaces, which validates the theoretical claim. Therefore, in practice, the theoretical claim that a single interface suffices with $O(\log n)$ channels is reasonably accurate, with the caveat that additional interfaces may be useful in simplifying protocol design and hiding switching delay.

In summary, in this report we have derived the lower and upper bounds on the capacity of static multichannel wireless networks. We have considered wireless networks having c channels, and $m \leq c$ interfaces per node. Each interface is capable of selecting appropriate transmission power, and lower bound constructions require global knowledge. Under this model, we have shown that in an arbitrary network, there is a loss in the network capacity when the number of interfaces per node is smaller than the number of channels. However, we have shown that in a random network, a single interface may suffice for utilizing multiple channels, as long as the number of channels is scaled as $O(\log n)$. We have then considered the impact of non-zero interface switching delay on capacity, and shown that in a random network with up to $O(\log n)$ channels, interface switching delay has no impact on capacity, provided each node is provisioned with a few extra interfaces. We have also considered the scenario where after an initial channel assignment, interfaces are not allowed to switch channels. Under this model, we show that if each node has only one interface, then there is a loss in network capacity. However, the capacity loss can be eliminated by providing an additional interface at each node, which shows that it may be possible to develop protocols which do not require interface switching, albeit at the cost of using extra hardware.

APPENDIX I

MAXIMUM NUMBER OF FLOWS PASSING THROUGH A CELL

Recall that each cell (see Figure 5) has an area $a(n)$. The unit square region is divided by the square grid into $\frac{1}{\sqrt{a(n)}}$ rows and columns. The proofs here use the following versions of Chernoff bound:

Lemma 7: (Chernoff Upper Tail Bound [37]) Let $X_1, \ldots, X_n$ be independent Poisson trials, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^{n} X_i$. Then, for $0 < \beta \leq 1$:

$$Pr\left[X \geq (1+\beta)E[X]\right] \leq \exp\left(-\frac{\beta^2}{3}E[X]\right)$$

Lemma 8: (Chernoff Lower Tail Bound [37]) Let $X_1, \ldots, X_n$ be independent Poisson trials, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^{n} X_i$. Then, for $0 < \beta < 1$:

$$Pr[X \leq (1-\beta)E[X]] \leq \exp\left(-\frac{\beta^2}{2}E[X]\right)$$

We next bound the maximum number of nodes in any row (or column).

Lemma 9: When n nodes are randomly placed on a unit torus that is divided into $Y_r = \frac{1}{\sqrt{a(n)}}$ rows (or columns) with $a(n) > 0$, then $Pr[$ any row (or column) has $\geq 2n\sqrt{a(n)}$ nodes $] \leq \frac{1}{\sqrt{a(n)}} \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right)$ for sufficiently large n.

Proof: Recall that each node is placed uniformly at random on the unit torus. Therefore, since there are Y_r rows of equal size, a node has a probability $\frac{1}{Y_r}$ of being placed in a particular row. Consider some row i. Let X_{ij} be an indicator variable that is 1 if node j is placed in row i. Let $X_i = \sum_{j=1}^{n} X_{ij}$ be the total number of nodes that are placed in row i. Then, $E[X_i] = \frac{n}{Y_r} = n\sqrt{a(n)}$. By applying the Chernoff bound from Lemma 7 (with $\beta = 1$), we have

$$Pr\left[\text{ row } i \text{ has} \geq 2n\sqrt{a(n)} \text{ nodes }\right] \leq \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right)$$

Since there are Y_r rows, applying the union bound, we have

$$\begin{aligned} Pr[\text{ any row has} \geq 2n\sqrt{a(n)} \text{ nodes }] &\leq Y_r \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right) \\ &= \frac{1}{\sqrt{a(n)}} \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right) \end{aligned}$$

The bound for maximum number of nodes in a column can be similarly derived. ■

Lemma 5: (restated here)

When the row-column routing is used, and $a(n) > \frac{\log n}{n}$, the maximum number of flows that pass through any cell (including flows originating and terminating in the cell) is $O\left(n\sqrt{a(n)}\right)$, with high probability.

Proof: Using the row-column routing technique, the flows that pass through any cell can be divided into two groups; flows that are being routed along the row containing the cell, from some source node till the desired destination column is reached, and flows that are being routed along the column containing the cell toward some destination node.

Each node is the originator of exactly one flow. From Lemma 9, the probability any row has more than $2n\sqrt{a(n)}$ nodes is at most $\frac{1}{\sqrt{a(n)}}\exp\left(\frac{-1}{3}n\sqrt{a(n)}\right)$. Therefore, the probability of having more than $2n\sqrt{a(n)}$ flows along a row is at most $\frac{1}{\sqrt{a(n)}}\exp\left(\frac{-1}{3}n\sqrt{a(n)}\right)$. This probability is a decreasing function of $a(n)$. When $a(n) > \frac{\log n}{n}$, this probability (maximized for this range of $a(n)$ when $a(n) \to \frac{\log n}{n}$) is upper bounded by $\frac{\sqrt{n}}{\exp(\frac{1}{3}\sqrt{n}\sqrt{\log n})}\frac{1}{\sqrt{\log n}}$ and tends to 0 as $n \to \infty$. Therefore, this result implies that the number of flows along a row is bounded by $O\left(n\sqrt{a(n)}\right)$ *whp*.

Next, we bound the number of flows along a column. Consider some column i. Let N_i be a random variable representing the number of nodes in column i. Let X_i be the number of flows along column i. Then, conditioning on the value of N_i,

$$\begin{aligned}
Pr\left[X_i \geq 4n\sqrt{a(n)}\right] &= \sum_{n_i=1}^{n} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] \\
&= \sum_{n_i=1}^{\frac{1}{2}n\sqrt{a(n)}} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] \\
&\quad + \sum_{n_i=\frac{1}{2}n\sqrt{a(n)}+1}^{2n\sqrt{a(n)}-1} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] \\
&\quad + \sum_{n_i=2n\sqrt{a(n)}}^{n} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] \\
&\leq Pr\left[N_i \leq \frac{1}{2}n\sqrt{a(n)}\right] \\
&\quad + \sum_{n_i=\frac{1}{2}n\sqrt{a(n)}+1}^{2n\sqrt{a(n)}-1} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] \\
&\quad + Pr\left[N_i \geq 2n\sqrt{a(n)}\right]
\end{aligned} \tag{12}$$

The first and third terms in the above equation are simplified by noting $Pr[A|B=b]Pr[B=b] \leq Pr[B=b]$. Using the Chernoff upper tail bound (Lemma 7) with $\beta = 1$ (the proof technique is similar to Lemma 9), we have

$$Pr[N_i \geq 2n\sqrt{a(n)}] \leq \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right) \tag{13}$$

Similarly, using the Chernoff lower tail bound (Lemma 8) with $\beta = \frac{1}{2}$, we have

$$Pr[N_i \leq \frac{1}{2}n\sqrt{a(n)}] \leq \exp\left(\frac{-1}{8}n\sqrt{a(n)}\right) \tag{14}$$

For a given topology, let $N_i = n_i$. The number of flows along column i is equal to the number of source nodes that have chosen a node in column i as the destination. Since each source node chooses a destination uniformly at random, any source node has a probability $\frac{n_i}{n}$ of choosing a node in column i as its destination. Let X_{ij} be an indicator variable that is 1 if node j has picked a node in column i as its destination. Then, $X_i = \sum_{j=1}^{n} X_{ij}$. Therefore, $E[X_i] = n_i$. Using the Chernoff bound from Lemma 7 with $\beta = 1$, we have

$$Pr[X_i \geq 2n_i] \leq \exp\left(\frac{-1}{3}n_i\right) \tag{15}$$

In the range $\frac{1}{2}n\sqrt{a(n)} < N_i < 2n\sqrt{a(n)}$, for any $N_i = n_i$, $Pr[X_i \geq 4n\sqrt{a(n)}] \leq Pr[X_i \geq 2n_i]$ (because when $a < b, Pr[X_i \geq b] \leq Pr[X_i \geq a]$). Simplifying further using the same technique, for any $N_i = n_i$ in this range, $Pr[X_i \geq 2n_i] \leq Pr[X_i \geq n\sqrt{a(n)}]$ (because the smallest value of n_i in this range is $\frac{1}{2}n\sqrt{a(n)}$). Therefore, in the range $\frac{1}{2}n\sqrt{a(n)} < N_i < 2n\sqrt{a(n)}$ we have,

$$\begin{aligned}
Pr\left[X_i \geq 4n\sqrt{a(n)} \mid \frac{1}{2}n\sqrt{a(n)} < N_i < 2n\sqrt{a(n)}\right] &\leq Pr\left[X_i \geq 2N_i \mid \frac{1}{2}n\sqrt{a(n)} < N_i < 2n\sqrt{a(n)}\right] \\
&\leq Pr[X_i \geq 2(\frac{1}{2}n\sqrt{a(n)})] \\
&\qquad \text{Because } Pr\left[X_i \geq 2n_i\right] \text{ is a decreasing function of } n_i \\
&\leq \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) \quad \text{(using Equation 15)}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\sum_{n_i=\frac{1}{2}n\sqrt{a(n)}+1}^{2n\sqrt{a(n)}-1} Pr\left[X_i \geq 4n\sqrt{a(n)} \mid N_i = n_i\right] Pr\left[N_i = n_i\right] &\leq \sum_{n_i=\frac{1}{2}n\sqrt{a(n)}+1}^{2n\sqrt{a(n)}-1} \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) Pr\left[N_i = n_i\right] \\
&= \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) \sum_{n_i=\frac{1}{2}n\sqrt{a(n)}+1}^{2n\sqrt{a(n)}-1} Pr\left[N_i = n_i\right] \\
&\leq \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) * 1 \\
&= \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) \qquad (16)
\end{aligned}$$

Substituting the results from Equation 13, Equation 14, and Equation 16 into Equation 12 we have,

$$\begin{aligned}
Pr[X_i \geq 4n\sqrt{a(n)}] &\leq \exp\left(\frac{-1}{8}n\sqrt{a(n)}\right) + \exp\left(\frac{-1}{3}n\sqrt{a(n)}\right) + \exp\left(\frac{-1}{6}n\sqrt{a(n)}\right) \\
&\leq 3\exp\left(\frac{-1}{8}n\sqrt{a(n)}\right) \quad \text{since } n\sqrt{a(n)} > 1 \qquad (17)
\end{aligned}$$

Applying the union bound (there are a total of $\frac{1}{\sqrt{a(n)}}$ columns) and using Equation 17, we can show that for any column

$$Pr[\text{ any column has} \geq 4n\sqrt{a(n)} \text{ flows }] \leq \frac{1}{\sqrt{a(n)}} 3\exp\left(\frac{-1}{8}n\sqrt{a(n)}\right) \tag{18}$$

The above probability is a decreasing function of $a(n)$. Therefore, when $a(n) > \frac{\log n}{n}$, the above probability (maximized for this range of $a(n)$ when $a(n) \to \frac{\log n}{n}$) is upper bounded by $3\frac{\sqrt{n}}{\sqrt{\log n}} \exp\left(\frac{-1}{8}\sqrt{n \log n}\right)$. This probability tends to 0 as $n \to \infty$. Therefore, the number of flows routed along any column is bounded by

$O(n\sqrt{a(n)})$, with high probability. Combining the bounds for flows along rows and columns, when $a(n) > \frac{\log n}{n}$, the total number of flows in any cell is $O(n\sqrt{a(n)})$ *whp*.

■

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