Connectivity and Capacity of Multi-Channel Wireless Networks with Channel Switching Constraints

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***Abstract*— This paper argues for the need to address the issue of multi-channel network performance under constraints on channel switching. We present examples from emergent directions in wireless networking to motivate the need for such a study, and introduce some models to capture channel switching constraints. For some of these models, we study connectivity and capacity of a wireless network comprising n randomly deployed nodes, equipped with a single interface each, when there are $c = O(\log n)$ channels of equal bandwidth $\frac{W}{c}$ available. We consider an adjacent (c, f) channel assignment where a node may switch between f adjacent channels, but the adjacent channel block is randomly assigned. We show that the *per-flow* capacity for this channel assignment model is $\Theta(W\sqrt{\frac{f}{cn\log n}})$. We then show how the adjacent $(c,2)$ assignment maps to the case of untuned radios. We also consider a random (c, f) assignment where each node may switch between a pre-assigned random subset of f channels. For this model, we prove that *per-flow* capacity is $O(W\sqrt{\frac{p_{rnd}}{n\log n}})$ (where $p_{rnd} = 1-(1-\frac{f}{c})(1-\frac{f}{c-1})\ldots(1-\frac{f}{c-f+1})$) and $\Omega(W\sqrt{\frac{f}{cn\log n}})$.**

***Index Terms*— Multi-channel, switching constraints, connectivity, capacity, adjacent (c, f) assignment, random (c, f) assignment, detour-routing.**

I. Introduction

Earlier work on protocols for multi-channel wireless networks [1] has assumed that each node is capable of switching on all channels. This assumption may be challenged by emerging paradigms in wireless networking, such as envisioned large-scale deployment of extremely inexpensive wireless devices embedded in the environment, and dynamic spectrum access via cognitive radio. We briefly summarize some such scenarios:

- The need for low-cost, low-power radio transceivers to be used in inexpensive sensor nodes can give rise to many situations involving constrained switching. Hardware complexity (and hence cost), and/or power consumption may be significantly reduced if each node operates only in a small spectral range, and switches between a small subset of adjacent channels (e.g., if the transceiver uses an oscillator with limited tunability). However, if more spectrum is available than a single device can utilize, it may be possible at time of manufacture to lock different devices on to different frequency ranges. Also, potentially a transceiver may have an RF channel selector comprising a bank of switchable filters [2], from which it may select one to use for transmission/reception.
- In cognitive radio networks, given a multi-hop network of secondary users attempting to utilize unused spectrum, some channels may be locally unusable due to the presence of an active primary user in the vicinity.

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Thus, there is need to address the issue of multi-channel network performance in the presence of constraints on channel-switching, both in terms of determining how asymptotic transport capacity is affected by the constraints, and designing protocols for efficient channel-coordination, and data-transfer.

It has been proposed in [3] that extremely inexpensive wireless devices can be manufactured if it is possible to handle untuned radios whose operating frequency may lie randomly within some band. Also considered in [3] is the possibility that each device may have a small number of such untuned radios, and a random network coding based approach is proposed to relay information between a single source-destination pair. Some work on cognitive radio has addressed the issue of coordination in the face of restricted and variable channel availability at individual nodes due to active primary users [4], [5].

However, no formal theoretical models have been developed for the various types of switching constraints encountered in these previous works, and in other anticipated scenarios, and the impact of the constraints on network performance in a general multi-hop setting has not been quantified.

In this paper we present an initial foundation for this domain by introducing some models for constrained channel assignment, and exploring issues of connectivity and transport capacity for some of these models.

We consider an adjacent (c, f) channel assignment model, and show that the *per-flow* capacity for this case is

$\Theta(W\sqrt{\frac{f}{cn\log n}})$. We then use the results for this model to obtain asymptotic capacity results for untuned radios with random source-destination pairs. We also consider a random (c,f) assignment model. For this model, we prove that *per-flow* capacity is $O(W\sqrt{\frac{p_{rnd}}{n\log n}})$ (p_{rnd} is defined in Section XI) and $\Omega(W\sqrt{\frac{f}{cn\log n}})$. [1] We also briefly discuss a spatially correlated channel assignment model.

Due to paucity of space, we are only able to provide high-level proof sketches in this paper, and most lemmas/theorems are stated without proof. Please see [7] for detailed proofs.

II. Some Models for Constrained Channel Assignment

In this section we elaborate on some of the models for constrained channel assignment that we propose. These models assume that nodes possess only one interface each, there are c channels available, and all channels are orthogonal. However, they may potentially be extended to the case where multiple interfaces are available at each node[2].

A. Adjacent (c,f) Assignment

We introduce an assignment model wherein a node can switch between a set of f contiguous channels ($2 \leq f \leq c$). Thus, if the frequency band is divided into c channels numbered 1, 2, ..., c in order of increasing frequency, then, at manufacture/pre-deployment time, each node is assigned a block location i uniformly at random from $\{1,...,c-f+1\}$ and thereafter it can switch between the set $\{i,...,i+f-1\}$. This model is relevant when each individual node has a tranceiver with limited tunability, and thus may only switch between a small set of contiguous channels. It is also possible to establish a mapping between specific instances of this model, and the case of untuned radios (see Section X).

B. Random (c,f) Assignment

In this assignment model, a node is assigned a subset of f channels ($2 \leq f \leq c$) uniformly at random from the set of all possible channel subsets of size f. This model can capture situations where tiny low-cost sensor nodes may be equipped with a transceiver having a bank of f filters (e.g., such a design has been proposed in [2]). One can envisage scenarios where each filter operates on some random channel determined at time of manufacture.

C. Spatially Correlated Channel Assignment

In this model, a set of N pseudo-nodes is placed randomly in the network, in addition to the regular network nodes. Each pseudo-node is assigned a randomly chosen channel. All network nodes within a distance R of a pseudonode with assigned channel i are blocked from using channel i. This model captures channel unavailability due to an active primary user in the vicinity in cognitive radio networks, as well as situations where an external source of noise leads to poor channel quality in a certain region.

[1] We have recently obtained new results showing that capacity with random (c,f) assignment is $\Theta(W\sqrt{\frac{p_{rnd}}{n\log n}})$, for $c = O(\log n)$. Please see [6].

[2] In these models, we assume that $c \geq 2$, as $c = 1$ is the single channel case in which $f = c = 1$ is the only possibility. In Section VI, we explain why we do not allow $f = 1$ for $c \geq 2$.

III. Network Model

In the assumed network model, n nodes are located uniformly at random in a unit area toroidal region. Nodes use a common transmission range $r(n)$. Interference is modeled using the Protocol Model [8]. There are c available channels of bandwidth $\frac{W}{c}$ each. We focus on the case where the total number of available channels $c = O(\log n)$. This is justifiable because in large scale deployments, the number of nodes will typically be much larger than the number of available channels. Besides, when $c = \omega(\log n)$, there is a large capacity degradation even with unconstrained channel switching (as shown in [1]), thus making channelization an increasing liability, and constrained switching may lead to additional degradation, and potentially unacceptable performance. As in [8], each node is source of exactly one flow. It chooses a point uniformly at random (we shall refer to these points as *pseudo-destinations* throughout this paper), and selects the node (other than itself) lying closest to that point as its destination.

IV. Notation and Terminology

We use standard asymptotic notation [9]. When $f(n) = O(g(n))$, any function $h(n) = O(f(n))$ is also $O(g(n))$. We often refer to such a situation as $h(n) = O(f(n)) \implies O(g(n))$. We often refer to results as holding *with high probability* (w.h.p.), by which we mean with probability 1 as $n \to \infty$. As in [8], we say that the per flow network throughput is $\lambda(n)$ if each flow in the network can be guaranteed a throughput of at least $\lambda(n)$ with probability 1 as $n \to \infty$. Whenever we use log without explicitly specifying the base, we imply the *natural* logarithm.

V. Related Work

It was shown by Gupta and Kumar [8] that for a single-channel single-interface scenario, in an arbitrary network, the per flow capacity scales as $\Theta(\frac{W}{\sqrt{n}})$ bit-m/s per flow, while in a random network, it scales as $\Theta(\frac{W}{\sqrt{n\log n}})$ bits/s. It was also shown in [8] that if the available bandwidth W is split into c channels, with each node having a dedicated interface per channel, the results remain the same.

The throughput-delay trade-off was studied in [10], and it was shown that the optimal trade-off is given by $D(n) = \Theta(nT(n))$ where $D(n)$ is delay, and $T(n)$ is throughput. The capacity of ultra-wideband (UWB) networks was studied in [11], and [12].

In the multi-channel context, an interesting scenario arises when the number of interfaces m at each node may be smaller than the number of available channels c. This issue was analyzed in [1] and it was shown that the capacity results are a function of the channel-to-interface ratio $\frac{c}{m}$. It was also shown that in the random network case, there are three distinct capacity regions: when $\frac{c}{m} = O(\log n)$, the per-flow capacity is

$\frac{W}{\sqrt{n\log n}}$, when $\frac{c}{m} = \Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the per flow capacity is $\Theta(W\sqrt{\frac{m}{nc}})$, and when $\frac{c}{m} = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the per-flow capacity is $\Theta(\frac{Wm\log\log n}{\log n})$.

Another relevant body of work is that on bond percolation in wireless networks, e.g. [13]. The constrained assignments considered by us also lead to nodes within range being able to communicate only with a certain probability. However, unlike percolation, in our case the probabilities are not independent for all nodes pairs.

A multi-channel multi-hop network architecture has been considered in [14] in which each node has a single transceiver, and nodes have a *quiescent* channel to which they tune when not transmitting. A node wishing to communicate with a destination tunes to its quiescent channel, and transmits the packet to a neighbor whose quiescent channel is the same as that of the destination. Thereafter, the packet proceeds towards the destination on the quiescent channel. This has some similarity to our model and constructions in that a flow seeks to transition to a target destination channel (see Sections IX and XIII for our constructions). However, in their case, the transition can happen trivially at the very first hop, since the source node is always capable of tuning to the destination's quiescent channel. In our models nodes can only switch on some channels, and this needs to be taken into account.

VI. Upper Bounds on Capacity

Some general constraints on the capacity of the network (for any channel assignment model) are as follows:

a) Source-Destination Constraint for $f = 1$: If $f = 1$, but $c > 1$, then a source and its destination should have the same channel for communication between them to be possible. This may not always happen if the channels are assigned randomly. To illustrate, consider the class of assignment models where the assignment to individual nodes is i.i.d. Suppose, $Pr[$i and $dst(i)$ share a channel $] \leq p$. If the traffic model is such that any single node can be the destination of only upto $D(n)$ flows, then we argue thus:

We can obtain at least $\lfloor\frac{n}{2D(n)}\rfloor$ pairs with distinct nodes (thus leading to independent probabilities). The probability that at least one of the n source-destination pairs have different channels can be lower bounded by the probability that at least one of these distinct pairs do not share a common channel, and this is at least $1 - p^{\lfloor\frac{n}{2D(n)}\rfloor}$. When $\log\left(\frac{1}{p}\right) = \omega(\frac{2D(n)}{n})$, it grows to 1, as $n \to \infty$. Thus, the network capacity would be 0. For the adjacent (c, f) and random (c, f) assignments studied in this paper, this condition holds when $c > 1$, and so $f = 1$ when $c > 1$ yields zero capacity. When $f > 1$, as in the rest of this paper, this constraint does not apply.

b) Connectivity Constraint: Suppose the necesary condition for connectivity is that $r(n) = \Omega(g(n))$. Thus, the spatial re-use in the network is limited to $O(\frac{1}{(g(n))^2})$ concurrent transmissions on any single channel. Besides, each source-destination is separated by average $\Theta(1)$ distance (see [8] for details) and hence average $\Theta(\frac{1}{r(n)})$ hops. Thus per flow throughput is limited to $O(\frac{W}{nr(n)})$.

c) Interference and Destination Bottleneck Constraint: In [1], it was established that the per flow capacity is constrained to $O(W\sqrt{\frac{1}{cn}})$, when single-interface nodes can switch to any channel. It was also shown that if some node can be the destination of upto $D(n)$ flows, the per-flow throughput is constrained to be $O(\frac{W}{D(n)})$. These upper bounds also apply to the adjacent (c, f)-assignment case, since whatever is achievable with adjacent (c, f) assignment, is also achievable when nodes can switch to any channel.

Note that since we are only interested in the region $c = O(\log n)$, the connectivity constraint is asymptotically dominant.

VII. Adjacent (c, f) Channel Assignment

Recall that in this model, the frequency band is divided into c channels numbered 1, 2, ..., c in order of increasing frequency, but an individual node can only use f channels (where $2 \leq f \leq c$). At deployment time, each node is assigned a block location i uniformly at random from $1, \ldots, c-f+1$ and thereafter it can switch between the set $i, \ldots, i+f-1$. Thus, the probability that a node is capable of switching to channel i is given by $p_s^{adj}(i) = \frac{\min\{i, c-i+1, f, c-f+1\}}{c-f+1}$, since channel i occurs in $\min\{i, c-i+1, f, c-f+1\}$ blocks, and each block is randomly chosen with probability $\frac{1}{c-f+1}$.

Let us call channels with $p_s^{adj}(i) \geq \frac{f}{2c}$ the *preferred* channels. Then, one can see that, for any set of f contiguous channels, at least $\lceil\frac{f}{2}\rceil$ of the channels have $p_s^{adj}(i) \geq \frac{f}{2c}$. Hence, each node can switch on $x \geq \lceil\frac{f}{2}\rceil \geq \frac{f}{2}$ *preferred* channels. Also note that non-preferred channels only occur at the fringes of the frequency band.

The probability that a node with block location i shares a channel with another randomly chosen node is given by $p_{adj}(i) = \frac{(1+\min\{i-1, f-1\}+\min\{c-f+1-i, f-1\})}{c-f+1}$. Since block locations are chosen uniformly at random from $1, \ldots, c-f+1$, the probability that two randomly chosen nodes share at least one channel is given by:

$$p_{adj} = \frac{1}{c-f+1}\sum_{i=1}^{c-f+1} p_{adj}(i) \qquad (1)$$

It can be seen that $\frac{\min\{f, c-f+1\}}{c-f+1} \leq p_{adj}(i) \leq \frac{\min\{2f-1, c-f+1\}}{c-f+1}$. Thus, $\frac{\min\{f, c-f+1\}}{c-f+1} \leq p_{adj} \leq \frac{\min\{2f-1, c-f+1\}}{c-f+1}$.

A. Necessary Condition for Connectivity

An adaptation of the proof techniques used to obtain the necessary condition for connectivity in [15], enables one to handle connectivity with adjacent (c, f) assignment.

Theorem 1: With an adjacent (c, f) channel assignment (when $c = O(\log n)$), if $p = \min\{\frac{2f-1}{c-f+1}, 1\}$, and $\pi r^2(n) = \frac{(\log n + b(n))}{pn}$, where $b = \lim_{n\to\infty} b(n) < +\infty$ then:

$$\lim_{n\to\infty}\inf Pr[\text{ disconnection }] \geq e^{-b}(1-e^{-b}) > 0$$

where by *disconnection* we imply the event that there is a partition of the network.
The proof is omitted due to space constraints. Please see [7].

B. Sufficient Condition for Connectivity

It can be shown that setting $r(n) = a_1\sqrt{\frac{c\log n}{fn}}$, for some suitable constant a_1, suffices for connectivity. This will be evident from our lower bound construction for capacity, and the proof is hence not presented separately.

VIII. Adjacent (c, f) Assignment: Capacity Upper Bound

We proved that the necessary condition for connectivity implies $r(n) = \Omega(\sqrt{\frac{c\log n}{fn}})$. Then by the connectivity constraint mentioned in Section VI, the per flow throughput is limited to $O(W\sqrt{\frac{f}{cn\log n}})$ (recall that, as in [15], the disconnection events considered involved individual nodes getting isolated, and thus some source node would be unable to communicate with its destination).

IX. Adjacent (c, f) Assignment: Capacity Lower Bound

We present a constructive proof that achieves $\Omega(W\sqrt{\frac{f}{cn\log n}})$. This construction has similarity to the constructions in [8], [10], and [1], but must now handle the constraint that a node may not switch on all channels. The surface of the unit torus is divided into square cells of area $a(n)$ each. The transmission range $r(n)$ is set to $\sqrt{8a(n)}$, thereby ensuring that any node in a given cell is within range of any other node in any adjoining cell. Since we utilize the *Protocol Model* [8], a node C can potentially interfere with an ongoing transmission from node A to node B, only if $BC \leq (1+\Delta)r(n)$. Thus, a transmission by A in a given cell can only be affected by transmissions in cells with some point within a distance $(2+\Delta)r(n)$ from it, and all such cells must lie within a circle of radius $O((1+\Delta)r(n))$. Since Δ is independent of n, the number of cells that interfere with a given cell is only some constant (say β).

We choose $a(n) = \frac{100c\log n}{fn}$ (i.e. $r(n) = \sqrt{\frac{800c\log n}{fn}}$).

Lemma 1: Suppose we are given a unit toroidal region with n points located uniformly at random, and the region is sub-divided into axis-parallel square cells of area $a(n)$ each. If $a(n) = \frac{100\alpha(n)\log n}{n}, 1 \leq \alpha(n) \leq \frac{n}{100\log n}$, then each cell has at least $100\alpha(n)\log n - 50\log n \geq 50\alpha(n)\log n$ points and at most $100\alpha(n)\log n + 50\log n \leq 150\alpha(n)\log n$ points, with probability at least $1 - \frac{50\log n}{n}$.

Thus, by Lemma 1, the number of nodes in any cell lies between $\frac{50c\log n}{f}$ and $\frac{150c\log n}{f}$ with probability at least $1 - \frac{50\log n}{n}$.

Lemma 2: If there are at least $\frac{50c\log n}{f}$ nodes in every cell D, then there are at least $12\log n$ nodes in each cell on each of the *preferred* channels, with probability at least $1 - q_1$, where $q_1 = O(\frac{1}{n^2})$.

Lemma 3: If there are at least $\frac{50c\log n}{f}$ nodes in every cell D, then, for all adjacent *preferred* channels i and $i+1$, there are at least $12\log n$ nodes in the cell having both channels i and $i+1$, with probability at least $1 - q_2$, where $q_2 = O(\frac{1}{n^2})$.

Lemma 4: If there are at least $\frac{50c\log n}{f}$ nodes in every cell, and if i and $i+x$ are both *preferred* channels, where $x \leq \lfloor\frac{f}{2}\rfloor$, then there are at least $12\log n$ nodes in the cell having both channels i and $i+x$, with probability at least $1 - q_3$, where $q_3 = O(\frac{1}{n^2})$.

A. Routing

Let us denote the source of a flow as S, the pseudo-destination as D', and the actual destination as D. If there were no constraints on switching, we could have used a routing strategy similar to that in [8], in which a flow traverses the cells intersected by the straight line SD', and thereafter needs to take at most one extra-hop to reach the actual destination D, which must necessarily lie either in the same cell as D' or in one of the 8 adjacent cells. If that were the case, it can be claimed that:

Lemma 5: The number of SD'D routes that traverse any cell is $O(n\sqrt{a(n)})$.
We shall hereafter refer to this routing as straight-line routing, as it basically comprises a straight-line except for the last hop.

Lemma 6: No node is the destination of more than $O(\log n) \implies O(na(n))$ flows.

For adjacent (c, f) assignment, we cannot stipulate that *all* flows be routed along the (almost) straight-line path SD'D. This is because the flow is required to traverse a minimum number of hops to be able to guarantee that it can switch from source channel to destination channel w.h.p. We elaborate further on this issue.

Channel Selection and Transition Strategy: Initially, after each source has chosen a random destination, the flows are processed in turn and each is assigned an initial source channel, as well as a target destination channel.

Suppose the source S of a flow is assigned channel set $(i, \ldots, i+f-1)$, while the destination D has $(j, \ldots, j+f-1)$. The flow chooses one of the $x \geq \frac{f}{c}$ *preferred* channels available at the source uniformly at random. Let us denote it by l. It also chooses one of the $y \geq \frac{f}{2}$ *preferred* channels available at the destination (let us call it r) as the channel on which the flow reaches the destination. The destination channel choice may be made in any manner, e.g. we may make an i.i.d. choice amongst all channels available at the destination. We assume, without loss of generality, that $l \leq r$. Suppose $r - l = k'\lfloor\frac{f}{2}\rfloor + m(0 \leq m < \lfloor\frac{f}{2}\rfloor)$. Thus $k' = \frac{r-l-m}{\lfloor\frac{f}{2}\rfloor} \leq \frac{c-1}{\frac{f-1}{2}} = \frac{2(c-1)}{f-1} \leq \frac{4c}{f}$. Note that given two preferred channels l and r all channels $l \leq i \leq r$ must also necessarily be preferred. Then, from Lemma 4, it is always possible to transition from l to r in at most $k'+1 \leq$ steps: $l \to l+\lfloor\frac{f}{2}\rfloor, l+\lfloor\frac{f}{2}\rfloor \to l+2\lfloor\frac{f}{2}\rfloor, \ldots, l+k'\lfloor\frac{f}{2}\rfloor \to l+k'\lfloor\frac{f}{2}\rfloor + m = r$. Thus, the route passes through a sequence of nodes $x_1, x_2, \ldots x_k$ such that x_1 and x_2 share channel l, x_2 and

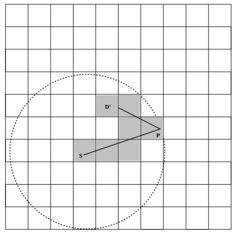

Fig. 1. Illustration of detour routing

x_3 share channel $l+\lfloor \frac{f}{2} \rfloor$ and so on. When $l \geq r$, the transitions are of the form $l \rightarrow l-\lfloor \frac{f}{2} \rfloor, \ldots, r$.

Thus, we stipulate that the straight-line path be followed if either the chosen source and destination channels are the same, or if the straight-line segment SD' comprises $h \geq \frac{4c}{f}$ *intermediate* hops. If S and D' (hence also D) lie close to each other, the hop-length of the straight line cell-to-cell path can be much smaller. In this case, a *detour* path is chosen. Consider a circle of radius $\frac{4c}{f} r(n)$ centered at S. Choose a point on this circle, say P. In the considered $c = O(\log n)$ regime, P can be any point on the circle. Then the route is obtained by traversing cells along SP and then PD'D. This ensures that the route has at least the minimum required hop-length (provided by segment SP). This situation is illustrated in Fig. 1.

A non-detour-routed flow is initially in a *progress-on-source-channel* mode, and keeps to the source channel till there are only $\frac{4c}{f}$ intermediate hops left to the destination. At this point, it enters *transition* mode, and starts making channel transitions along the remaining hops, till it has transitioned into its chosen destination channel. Thereafter, it remains on that channel. When a flow enters a cell in *progress-on-source-channel* mode, amongst all nodes in that cell capable of switching on that channel, it is assigned to the node which has the least number of flows assigned to it on that channel so far.

A detour-routed flow is always in *transition* mode.

Lemma 7: Given that the high probability event in Lemma 4 holds, suppose a flow is on *preferred* source channel i and needs to finally be on *preferred* destination channel j. Then after having traversed $h \geq \frac{4c}{f}+1$ cells (recall that $2 \leq f \leq c$) , it is guaranteed to have made the transition.

Lemma 8: The length of any route increases by only $O(\frac{c}{f}) \implies O(\log n)$ hops due to detour routing. The average route length increases by $O(\log n)$ hops.

Lemma 9: If the number of distinct flows traversing any cell is x with pure straight-line routing, it is $x+O(n\frac{c^2}{f^2}r^2(n)) \implies x+O(\log^4 n)$ even with detour routing.

Lemma 10: The number of distinct flows traversing any cell is $O(n\sqrt{a(n)})$ w.h.p. even with detour routing.

Lemma 11: The number of flows traversing any cell in *transition* mode is $O(\log^4 n)$ w.h.p.

B. Balancing Load within a Cell

Per-Channel Load: Recall that each cell has $O(na(n))$ nodes w.h.p., and $O(n\sqrt{a(n)})$ flows traversing it w.h.p.

Lemma 12: The number of flows that enter any cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Lemma 13: The number of flows that leave any given cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Per-Node Load:

Lemma 14: The number of flows that are assigned to any one node in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

C. Transmission Schedule

As noted earlier, each cell can face interference from at most a constant number β of nearby cells. Thus, if we consider the resultant cell-interference graph, it has a chromatic number at most $1+\beta$. We can hence construct a global schedule having $1+\beta$ unit time slots in each round. In any slot, if a cell is active, then all interfering cells are inactive. The next issue is that of intra-cell scheduling. We need to schedule transmissions during the cell's slot, so as to ensure that at any time instant, there is at most one transmission on any given channel in the cell. Besides, we also need to ensure that no node is expected to transmit or receive more than one packet at any time instant. We use the following procedure to obtain an intra-cell schedule:

We construct a conflict graph based on the nodes in the active cell, and its adjacent cells (note that the hop-sender of each flow shall lie in the active cell, and the hop-receiver shall lie in one of the adjacent cells), as follows: we create a separate vertex for each flow that requires a hop-transmission in the cell (note that we counted possible repeat traversals by detour-routed flows separately in Lemma 11, and now a twice-traversal can be treated like two distinct flows for scheduling purposes). Since the flow has an assigned channel on which it operates in that particular hop, each vertex in the graph has an implicit associated channel. Besides, each flow (and hence its vertex) has an associated pair of nodes corresponding to the hop-endpoints. Two vertices are connected by an edge if (1) they have the same associated channel, or (2) at least one of their associated nodes is the same. The scheduling problem thus reduces to obtaining a vertex-coloring of this graph. If we have a vertex coloring, then it ensures that (1) a node is never simultaneously sending/receiving for more than one flow (2) no two flows on the same channel are active simultaneously. The number of neighbors of a graph vertex is upper bounded by the number of flows entering/leaving the active cell on that channel, and the number of flows assigned to the flow's two hop endpoints (both hop-sender and hop-receiver). Thus, it can be seen from Lemmas 12, 13 and 14 that the degree of the conflict graph is $O(\frac{n\sqrt{a(n)}}{c})$. Since any graph with maximum degree d is vertex-colorable in at most $d+1$ colors, the conlict graph can be colored in $O(\frac{n\sqrt{a(n)}}{c})$ colors.

Thus the cell-slot is divided into $O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{\sqrt{\frac{cn\log n}{f}}}{c})$ equal length subslots, and all flows in the cell get a slot for transmission. This yields that each flow will get $\Omega(W\sqrt{\frac{f}{cn\log n}})$ throughput.

We thus obtain the following theorem:

Theorem 2: With an adjacent (c, f)-channel assignment, the network capacity is $\Theta(W\sqrt{\frac{f}{cn\log n}})$ per flow.

X. THE CASE OF UNTUNED RADIOS

The untuned channel model is as follows: each node possesses a transceiver with carrier frequency uniformly distributed in the range (F_1, F_2), and admits a spectral bandwidth B. Let $c = \lfloor \frac{F_2 - F_1}{B} \rfloor$. Then c is the maximum number of disjoint channels that could be possible. However the channels are untuned and hence partially ovelapping, rather than disjoint. As per the assumption in [3], two nodes can communicate directly if the carrier frequency of one is admitted by the other, i.e., if there is at least 50% overlap between two channels, communication is possible. We consider the issue of capacity of a randomly deployed network of n nodes, where each node has an untuned radio, and each node is the source of one flow, with a randomly chosen destination.

Even though each node only possesses a single radio and stays on a single sub-band, due to the partial overlap between sub-bands, it is still possible to ensure that any pair of nodes will be connected via some path. Contrast this to the case of orthogonal channels, where we argued in Section VI that when $f = 1$, and $c > 1$, some pairs of nodes are disconnected from each other because they do not share a channel. It is possible to map the partial overlap feature of the untuned channel case to adjacent $(2c+2, 3)$ and $(4c+1, 2)$ assignment. Note that $f = 2$ allows for all nodes to be connected, even with orthogonal channels.

We map the untuned radio scenario to a scenario having $(2c+2, 3)$ adjacent channel assignment.

We perform a virtual channelization of the band (F_1, F_2) into $2c$ orthogonal sub-bands. We add an additional (virtual) sub-band of the same width at each end of the band, to get $2c+2$ orthogonal channels, numbered $1, \ldots, 2c+2$. Thus 1 and $2c+2$ are the artificially added channels. If a radio's carrier frequency lies within virtual channel i, it is associated with virtual channel block $(i-1, i, i+1)$, and $i-1$ is called its primary virtual channel. Thus the primary channel can only be one of $1, 2, \ldots, 2c$ (since the carrier frequency can only fall in $2, .., 2c+1$). If a node's primary channel is i, it is capable of communicating with all nodes with primary virtual channel $i-2 \leq j \leq i+2$ in the virtual channelization. In the actual situation, the node with the untuned radio would be able to communicate with some subset of those nodes. Thus, if a pair of nodes cannot communicate directly in the virtual channelization, they cannot do so in the actual situation either, and disconnection events in the former are preserved in the latter. The probability that a node has virtual channel block $(j, j+1, j+2)$ is $\frac{1}{2c}$, i.e., the same as for adjacent $(2c+2, 3)$ assignment, and the necessary condition for the (virtual) $(2c+2, 3)$ assignment continues to hold for the corresponding untuned radio case. This yields an upper bound on capacity of $O(W\sqrt{\frac{1}{cn\log n}})$.

It can be shown that a schedule constructed for an adjacent $(4c+1, 2)$ assignment can be used almost as-is with untuned radios (except that the number of subslots in the cell-slot must increase by a factor of 11 to avoid interference due to overlap).

We perform a virtual channelization of the band (F_1, F_2) into $4c+1$ orthogonal sub-bands. If a radio's carrier frequency lies within virtual channel i, it is associated with virtual channel block $(i, i+1)$, and i is called its primary virtual channel. Note that if a node's primary channel is i, it is always capable of communicating with all nodes with primary virtual channel $i-1 \leq j \leq i+1$, but we will pretend that it can only communicate with those having i or $i+1$. Thus, if a pair of nodes share a channel in the virtual channelization, then they are always capable of direct communication in the actual untuned radio situation. The probability that a radio has virtual channel block $(i, i+1)$ is $\frac{1}{4c}$, same as for adjacent $(4c+1, 2)$ assignment. In the adjacent $(4c+1, 2)$ assignment, all channel are orthogonal and can operate concurrently. With untuned radios, we assume two nodes can interfere if there is some spectral overlap. Thus, a transmission by a node on carrier frequency F can interfere with transmissions by nodes with carrier frequency in the range $(F-B, F+B)$. Hence, the transmission schedule for untuned radios is made to follow the additional constraint that if a node with primary virtual channel i is active then no node with primary channel $i-5 \leq j \leq i+5$ should be active simultaneously. This would decrease capacity by a factor of 11, but would not affect the order of the asymptotic results. Also, in the actual network involving untuned radios, a transceiver can use upto $B = \frac{F_2 - F_1}{c}$ spectral bandwidth, while in the adjacent $(4c+1, 2)$ case, it would be $\frac{F_2 - F_1}{4c+1}$, leading to the possibility of having a higher data-rate in the former, given the same transmission power, modulation, etc. However this can only affect capacity by a small constant factor, which does not affect the order of the results.

In the adjacent $(4c+1, 2)$ case, our construction performs transitions to ensure that a source on channels $(i, i+1)$ and a destination on channels $(i+j, i+j+1)$ can communicate. In the untuned radio case, transitioning is done through nodes that provide the required virtual channel pair, and the same transition strategy as for $(4c+1, 2)$ assignment continues to work. Hence the capacity is $\Omega(W\sqrt{\frac{1}{cn\log n}})$ per flow.

We re-emphasize that even though $f = 1$, the untuned nature of the radios allows for a progressive shift in the frequency over which the packet gets transmitted, thereby allowing a step-by-step transition from the source's carrier frequency to a frequency admitted by the destination. The adjacent (c, f) model captures this progressive frequency-shift characteristic, and is thus able to model the untuned radio situation.

From the upper and lower bounds proved in this section, it follows that the capacity of the untuned radio network, when

$c=O(\log n)$, is $\Theta(W\sqrt{\frac{1}{cn\log n}})$ per flow.

XI. Random (c,f) assignment

In this assignment model, a node is assigned a subset of f channels uniformly at random from the set of all possible channel subsets of size f. Thus the probability that a node is capable of switching on a given channel i is $p_s^{rnd}(i)=\frac{f}{c}=p_s^{rnd},\forall i$, and the probability that two nodes share at least one channel is given by $p_{rnd}=1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$.

A. Necessary Condition for Connectivity

Theorem 3: With a random (c,f) channel assignment (when $c=O(\log n)$), if $\pi r^2(n)=\frac{(\log n+b(n))}{pn}$, where $p=p_{rnd}=1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$, and $c=O(\log n)$, and $b=\lim_{n\to\infty}b(n)<+\infty$ then:

$$\lim_{n\to\infty}\inf Pr[\text{ disconnection }]\geq e^{-b}(1-e^{-b})>0$$

where by *disconnection* we imply the event that there is a partition of the network.

B. Sufficient Condition for Connectivity

Theorem 4: With random (c,f) assignment (when $c=O(\log n)$), if $\pi r^2(n)=\frac{800\pi\log n}{p_{rnd}n}$, then:

$$Pr[\text{ network is connected }]\to 1$$

Proof: We present a construction based on a notion of per-node backbones. Consider a subdivision of the toroidal unit area into square cells of area $a(n)=\frac{100\log n}{p_{rnd}n}$. Then by setting $\alpha(n)=\frac{1}{p_{rnd}}$ in Lemma 1 there are at least $\frac{50\log n}{p_{rnd}}$ nodes in each cell with high probability. Set $r(n)=\sqrt{8a(n)}$. Then a node in any given cell has all nodes in adjacent cells within its range. Within each cell, choose $\frac{2\log n}{p_{rnd}}$ nodes uniformly at random, and set them apart as *transition facilitators* (the meaning of this term shall become clear later). This leaves at least $\frac{48\log n}{p_{rnd}}$ nodes in each cell that can act as *backbone candidates*.

Consider any node in any given cell. The probability that it can communicate to any other random node in its range is p_{rnd}. Then the probability that in an adjacent cell, there is no backbone candidate node with which it can communicate is less than $(1-p_{rnd})^{\frac{48\log n}{p_{rnd}}}\leq\frac{1}{e^{48\log n}}=\frac{1}{n^{48}}$. The probability that a given node cannot communicate with any node in some adjacent cell is thus at most $\frac{8}{n^{48}}$ (as there are upto 8 adjacent cells per node). By applying the union bound over all n nodes, the probability that at least one node is unable to communicate with any backbone candidate node in at least one of its adjacent cells is at most $\frac{8}{n^{47}}$.

We associate with each node x a set of nodes $\mathcal{B}(x)$ called the primary backbone for x. $\mathcal{B}(x)$ is constituted as follows. Throughout the procedure, cells that are already covered by the under-construction backbone are referred to as *filled* cells. x is by default a member of $\mathcal{B}(x)$, and its cell is the first *filled* cell. From each adjacent cell, amongst all backbone candidate nodes sharing at least one common channel with x, one is chosen uniformly at random is added to $\mathcal{B}(x)$. Thereafter, from each cell bordering a filled cell, of all nodes sharing at least one common channel with some node already in $\mathcal{B}(x)$, one is chosen uniformly at random, and is added to $\mathcal{B}(x)$; the cell gets added to the set of filled cells. This process continues iteratively, till there is one node from every cell in $\mathcal{B}(x)$. From our earlier observations, for all nodes x, $\mathcal{B}(x)$ eventually covers all cells with probability at least $1-\frac{8}{n^{47}}$.

Now consider any pair of nodes x and y. If $\mathcal{B}(x)\cap\mathcal{B}(y)\neq\phi$, i.e., the two backbones have a common node, then x and y are obviously connected, as one can proceed from x on $\mathcal{B}(x)$ towards one of the intersection nodes, and thence to y on $\mathcal{B}(y)$, and vice-versa.

Suppose, the two backbones are disjoint. Then x and y are still connected if there is some cell such that the member of $\mathcal{B}(x)$ in that cell (let us call it q_x) can communicate with the member of $\mathcal{B}(y)$ in that cell (let us call it q_y), either directly, or through a third node. q_x and q_y can communicate directly with probability 1 if they share a common channel. Thus the case of interest is one in which no cell has q_x and q_y sharing a channel. If they do not share a common channel, we consider the event that there exists a third node z amongst the *transition facilitators* in the cell through whom they can communicate.

Note that, for two given backbones $\mathcal{B}(x)$ and $\mathcal{B}(y)$, the probability that in a network cell, given q_x and q_y that do not share a channel, they can both communicate with a third node z that did not participate in backbone formation and is known to lie in the same cell, is independent across cells. Therefore, the overall probability can be lower-bounded by obtaining for one cell the probability of q_x and q_y communicating via a third node z, given they have no common channel, considering that each cell has at least $\frac{2\log n}{p_{rnd}}$ possibilities for z, and treating it as independent across cells. We elaborate this further.

Let q_x have the set of channels $C(q_x)=\{c_{x_1},...,c_{x_f}\}$, and q_y have the set of channels $C(q_y)=\{c_{y_1},...,c_{y_f}\}$, such that $C(q_x)\cap C(q_y)=\phi$. Consider a third node z amongst the transition facilitators in the same cell as q_x and q_y. We desire z to have at least one channel common with both $C(q_x)$ and $C(q_y)$. Then let us merely consider the possibility that z enumerates its f channels in some order, and then inspects the first two channels, checking the first one for membership in $C(q_x)$, and checking the second one for membership in $C(q_y)$. This probability is $\left(\frac{f}{c}\right)\left(\frac{f}{c-1}\right)>\frac{f^2}{c^2}$. Thus q_x and q_y can communicate through z with probability $p_z>\frac{f^2}{c^2}=\Omega(\frac{1}{\log^2 n})$. There are $\frac{2\log n}{p_{rnd}}$ possibilities for z within that cell, and all the possible z nodes have i.i.d channel assignments. Thus, the probability that q_x and q_y cannot communicate through any z in the cell is at most $(1-p_z)^{\frac{2\log n}{p_{rnd}}}$, and the probability they can indeed do so is $p_{xy}>1-(1-p_z)^{\frac{2\log n}{p_{rnd}}}$.

Thus, the probability that this happens in none of the $\frac{1}{a(n)}=\frac{p_{rnd}n}{100\log n}$ cells is at most $(1-p_{xy})^{\frac{p_{rnd}n}{100\log n}}<(1-p_z)^{\frac{2\log n}{p_{rnd}}\frac{p_{rnd}n}{100\log n}}<(1-\frac{1}{c^2})^{\frac{2\log n}{p_{rnd}}\frac{p_{rnd}n}{100\log n}}\to e^{-\Omega(\frac{n}{\log^2 n})}$ (recall that $c=O(\log n)$). Applying union bound over all $\binom{n}{2}<\frac{n^2}{2}$ node pairs, the probability that some pair of nodes are not connected is at most $\frac{n^2 e^{-\Omega(\frac{n}{\log^2 n})}}{2} < \frac{1}{2} e^{-\Omega(\frac{n}{\log^2 n})+2\log n} \to 0$. Thus the probability of a connected network converges to 1. ■

XII. Random (c, f) Assignment: Capacity Upper Bound

Since the necessary condition for connectivity requires that $r(n) = \Omega(\frac{\log n}{p_{rnd} n})$, the per flow capacity is $O(W\sqrt{\frac{p_{rnd}}{n \log n}})$ from the discussion on the connectivity upper bound in Section VI.

XIII. Random (c, f) Assignment: Capacity Lower Bound

We present a constructive proof that achieves $\Omega(W\sqrt{\frac{f}{cn\log n}})$. This construction is quite similar to that for adjacent (c, f) assignment. The surface of the unit torus is divided into square cells of area $a(n)$ each. The transmission range is set to $\sqrt{8a(n)}$, thereby ensuring that any node in a given cell is within range of any other node in any adjoining cell. As discussed for the adjacent assignment case, the number of cells that interfere with a given cell is only some constant (say β). We choose $a(n) = \frac{100c\log n}{fn}$ (resultantly $r(n) = \sqrt{\frac{800c\log n}{fn}}$). Thus, Lemma 1 applies for this case too.

Lemma 15: If there are $\frac{50c\log n}{f}$ nodes in every cell, then there are at least $25\log n$ nodes in each cell on each of the c channels, with probability at least $1-q$, where $q = O(\frac{1}{n^4})$.

A. Routing

Observe that Lemmas 5 and 6 stated in Section IX for SD'D routing are applicable here too.

In case of random (c, f) assignment, as with adjacent assignment, we cannot stipulate that *all* flows be routed along the straight-line path SD'D. A flow may be required to traverse a minimum number of hops to be able to ensure that it will find an opportunity to make the switch from source channel to destination channel.

Channel Selection and Transition Strategy: Initially, after each source has chosen a random destination, the flows are processed in turn and each is assigned an initial source channel, as well as a target destination channel. The source channel for a flow originating at node S is chosen according to the uniform distribution from the f channels available at S. The destination channel may be chosen from amongst the f channels available at destination D in any manner, e.g., it may be the one with the smallest number of incoming flows assigned to it so far.

We stipulate that a non-detour-routed flow is initially in a *progress-on-source-channel* mode, and keeps to the source channel till there are only $\lceil\frac{4c}{25f}\rceil$ intermediate hops left to the destination. At this point, it enters a *ready-for-transition* mode, and actively seeks opportunities to make a channel transition along the remaining hops. It makes use of the first opportunity that presents itself, i.e., if a node in a on-route cell provides the source-destination channel pair, the flow is assigned to that node for relaying (the node received it on the source channel, and forwards it on the destination channel). Once it has made the transition, it remains on the destination channel.

During the *progress-on-source-channel* phase, the next hop node is chosen to be the node in the next cell which has the smallest number of flows assigned so far on that channel, amongst all nodes that can switch on the source channel. In the *ready-for-transition* phase, it may be assigned to *any eligible node* that provides either the transition opportunity, or the source channel (for flows yet to find a transition), or the destination channel (for flows that have already transitioned into their destination channel).

A detour-routed flow is always in *ready-for-transition* mode.

Lemma 16: Suppose a flow is on source channel i and needs to finally be on destination channel j. Then after having traversed $h \geq \lceil\frac{2(c-1)}{(f-1)}\rceil$ distinct cells (recall that $2 \leq f \leq c$, an hence $h = O(\log n)$), it will have found an opportunity to make the transition w.h.p.

Note that $\frac{2(c-1)}{25(f-1)} \leq \frac{4c}{25f}$. Thus, the (almost) straight-line SD'D path is followed if either source and destination channels are the same, or if the straight-line segment SD' provides $h \geq \lceil\frac{4c}{25f}\rceil$ *intermediate* hops. If S and D' (hence also D) lie close to each other, the hop-length of the straight line cell-to-cell path can be much smaller. In this case, a *detour* path is chosen. Consider a circle of radius $\lceil\frac{4c}{25f}\rceil r(n)$ centered at S. Choose any point on this circle, say P, so long as P does not lie in the same cell as D (this guarantees at least one intermediate hop even if $\frac{4c}{25f} \leq 1$). Then the route is obtained by traversing cells along SP and then PD. This ensures that the route has at least the minimum required hop-length (since the segment SP always provides at heast $\lceil\frac{4c}{25f}\rceil$ distinct hops(cells). This situation is illustrated in Fig. 1.

Lemma 17: The number of distinct flows traversing any cell is $O(n\sqrt{a(n)})$ even with detour routing.

Lemma 18: The number of flows traversing any cell in *ready-for-transition* mode is $O(\log^4 n)$ w.h.p.

B. Balancing Load within a Cell

Per-Channel Load: Recall that each cell has $O(na(n))$ nodes w.h.p., and $O(n\sqrt{a(n)})$ flows traversing it w.h.p.

Lemma 19: The number of flows that enter any cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Lemma 20: The number of flows that leave any given cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Per-Node Load:

Lemma 21: The number of flows that are assigned to any one node in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

C. Transmission Schedule

The transmission schedule is obtained in a manner similar to Section IX-C. First, we obtain a global inter-cell schedule, and then construct a conflict graph for intra-cell scheduling. Thus, it can be seen from Lemmas 19, 20 and 21 that the degree of the conflict graph is $O(\frac{n\sqrt{a(n)}}{c})$. Thus the graph can be colored in $O(\frac{n\sqrt{a(n)}}{c})$ colors. Thus the cell-slot is divided into $O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{\sqrt{\frac{cn\log n}{f}}}{c})$ equal length subslots, and all traversing flows get a slot for transmission. This yields that each flow will get $\Omega(\sqrt{\frac{f}{cn\log n}}W)$ throughput. We thus obtain the following theorem:

Theorem 5: With a random (c, f) channel assignment, the described construction achieves throughput of $\Omega(W\sqrt{\frac{f}{cn\log n}})$ per flow.

XIV. Discussion

The lower bound constructions for the two assignment models yield interesting insights. As is intuitive, when all nodes cannot switch on all channels, the transmission range needs to be larger to preserve network connectivity, leading to a capacity degradation. Also, it may no longer be possible to use the shortest route towards the destination, and a flow may need to take a circuitous path (*detour routing*) in order to ensure that the destination is reached. However, when the number of channels is much smaller than the number of nodes, the increase in the length of the routes is not asymptotically significant. Taking all factors into account, when $c = O(\log n)$, given a sufficiently dense network, it is beneficial to attempt to use all channels by assigning different channel subsets to different nodes, rather than follow the naive approach of using the same f channels at all nodes. In the latter case, the per-flow capacity would be reduced to $\Theta(W\frac{f}{c\sqrt{n\log n}})$. Thus the *use-all-channels* approach outperforms the *f-common-channels* approach by a factor of $\sqrt{\frac{c}{f}}$. As an example, even when $f = 2$, utilizing all channels yields a capacity of the order of $\sqrt{c}$ channels. As mentioned earlier, we have recently obtained new results [6] showing that random (c, f) capacity is $\Theta(W\sqrt{\frac{p_{rnd}}{n\log n}})$, which converges much faster to the unconstrained capacity.

It is also to be noted that when $f = c$, our models reduce to the unconstrained switching model in [1] with a single interface per node. For this case, our per-flow capacity results yield $\Theta(\sqrt{\frac{W}{n\log n}})$, as also obtained in [1] for $\frac{c}{m} = O(\log n)$. However, we are able to achieve the optimal capacity by using a much simpler random flow-channel mapping. We also note that the techniques using random flow-channel assignment and detour routing, which were devised for the models in this paper, can be applied to other situations, e.g., the deterministic fixed assignment considered in [16].

Another interesting insight is yielded by the results for random (c, f) assignment. Note that a transmission range of $\Theta(\sqrt{\frac{\log n}{p_{rnd}n}})$ is both necessary and sufficient for connectivity. However, at this transmission range, it is possible that some cells may have some channels missing. Thus, the subgraph induced by a certain channel (obtained by retaining only nodes capable of switching on that channel, and assuming this is the only channel they can use) may not necessarily be connected, but the overall network graph is always connected at this transmission range. This may perhaps at times make it necessary (due to connectivity concerns) to schedule different links of a flow on different channels, even if the source and destination share a channel. Note that if we set $r(n) = \Theta(\sqrt{\frac{c\log n}{fn}})$, then a source-destination pair that share a channel always have a route with all links using that channel (though it is not capacity-optimal to use it with random (c, f) assignment), since each channel is available on some nodes in each cell.

XV. Conclusion

In this paper we have presented a case for the study of multi-channel networks with channel switching constraints. We introduced some models for channel switching constraints, and presented connectivity and capacity results for two such models, viz. adjacent (c, f) assignment, and random (c, f)-assignment, when $c = O(\log n)$. While originally derived for channelization in the frequency domain, our results can also be interpreted in the time domain, and provide insights about energy-capacity trade-offs in networks with low-duty-cycle nodes. Furthermore, we believe that there is significant potential for extension of the current models, as well as study of a wider range of switching constraints.

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